In[ ]:=

```
(* Meeting point: NL_NIL *)
ClearAll["Global`*"]
gradedForm /: MakeBoxes[gradedForm[poly_], form_] :=
  Module[{t}, With[{vars = Alternatives @@ Variables@poly},
    RowBox[Riffle[RowBox[{"(", MakeBoxes[#, form], ")"}] & /@
       CoefficientList[poly /. v : vars :> t * v, t], "+"]]]];
```

$$F[f1\_, f2\_, f3\_] := \begin{pmatrix} f1 & f2 & f1\ f2 \\ -f1 & f1 & f3 \\ -f2 & f3 & f2 \end{pmatrix}$$

```
g[f1_, f2_, f3_] :=
  Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] // gradedForm;
s = 5;
g1[f1_, f2_, f3_] := Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω];
g2[f1_, f2_, f3_] := f1 - f2;
Solve[g1[f1, f2, f3] == 0 && g2[f1, f2, f3] == 0, {f2, f3}]
```

$$\text{Solve}\left[-3 + 10 k^2 - 11 k^4 + 4 k^6 == 0, k\right]$$

$$\text{multSolveProj1}[f1\_] := \frac{1}{2}\left(f1 - f1^2 - f1\sqrt{-3 - 2 f1 + f1^2}\right);$$

$$\text{multSolveProj2}[f1\_] := \frac{1}{2}\left(f1 - f1^2 + f1\sqrt{-3 - 2 f1 + f1^2}\right);$$

```
Solve[multSolveProj1[f1] == a, f1]
Solve[multSolveProj2[f1] == a, f1]
```

$$\text{Simplify}\left[g\left[\frac{a - \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}, \frac{a + \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}, a\right]\right] \text{ // FullForm}$$

(* This shows that the guessing curve indeed on the contour *)

$$\text{Simplify}\left[g\left[f1, f1, \frac{1}{2}\left(f1 - f1^2 - f1\sqrt{-3 - 2 f1 + f1^2}\right)\right]\right] \text{ // FullForm}$$

$$\text{Simplify}\left[g\left[f1, f1, \frac{1}{2}\left(f1 - f1^2 + f1\sqrt{-3 - 2 f1 + f1^2}\right)\right]\right] \text{ // FullForm}$$

(* This shows that NL on the contour *)

Out[ ]=

$$\left\{\left\{f2 \to f1, f3 \to -f1 - 2\sqrt{2}\ f1 + f1^2\right\}, \left\{f2 \to f1, f3 \to -f1 + 2\sqrt{2}\ f1 + f1^2\right\},\right.$$
$$\left\{f2 \to f1, f3 \to \frac{1}{2}\left(f1 - f1^2 - f1\sqrt{-3 - 2 f1 + f1^2}\right)\right\},$$
$$\left.\left\{f2 \to f1, f3 \to \frac{1}{2}\left(f1 - f1^2 + f1\sqrt{-3 - 2 f1 + f1^2}\right)\right\}\right\}$$

Out[ ]=

$$\left\{\{k \to -1\}, \{k \to -1\}, \{k \to 1\}, \{k \to 1\}, \left\{k \to -\frac{\sqrt{3}}{2}\right\}, \left\{k \to \frac{\sqrt{3}}{2}\right\}\right\}$$

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[ ]=

$$\left\{\left\{f1 \to \frac{a - \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}\right\}, \left\{f1 \to \frac{a + \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}\right\}\right\}$$

Solve: There may be values of the parameters for which some or all solutions are not valid.

Out[ ]=

$$\left\{\left\{f1 \to \frac{a - \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}\right\}, \left\{f1 \to \frac{a + \sqrt{-3 a^2 - 4 a^3}}{2 (1 + a)}\right\}\right\}$$

Out[ ]//FullForm=

```
gradedForm[0]
```

Out[ ]//FullForm=

```
gradedForm[0]
```

Out[ ]//FullForm=

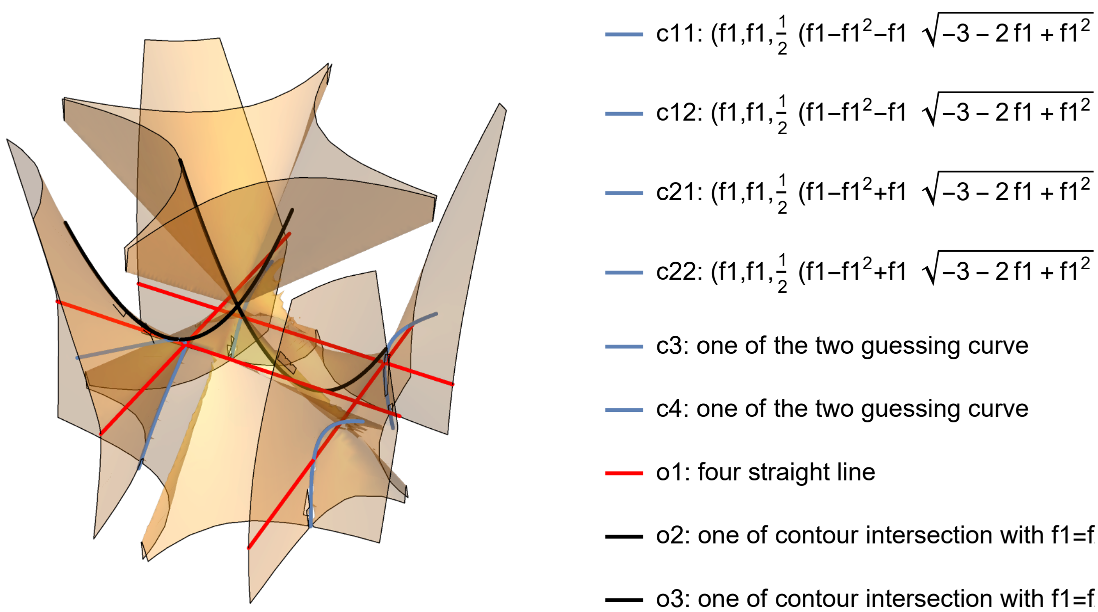
```
gradedForm[0]
```

In[ ]:=

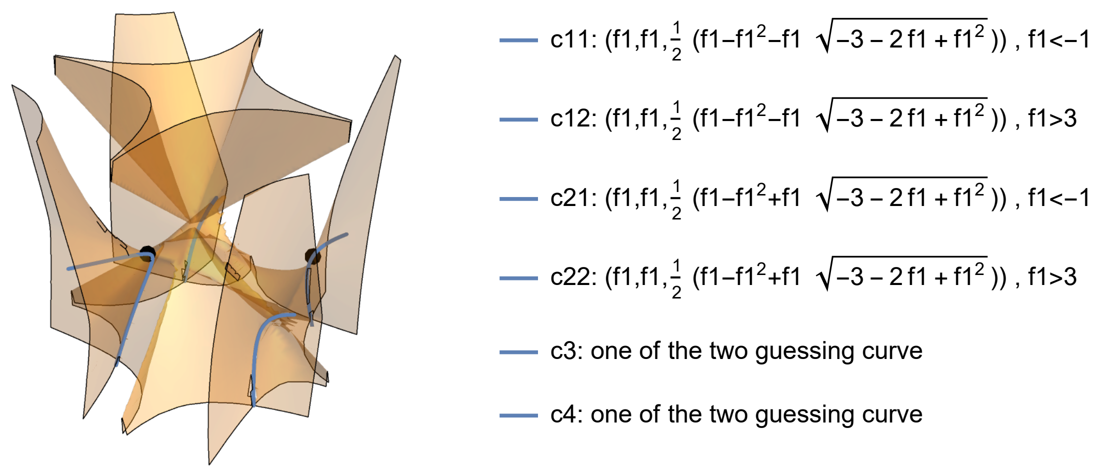
```
surface = ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
   {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, Mesh → None, ContourStyle → Opacity[0.3],
   AxesLabel → Automatic, Axes → False, Boxed → False];
c11 = ParametricPlot3D[{f1, f1, 1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2])},
   {f1, -3, -1}, (*PlotStyle→RGBColor[0,1,0],*)
   PlotLegends → {"c11: (f1,f1,1/2 (f1-f1^2-f1 Sqrt[-3 - 2 f1 + f1^2])) , f1<-1"}];
c12 = ParametricPlot3D[{f1, f1, 1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2])},
   {f1, 3, 3.3}, (*PlotStyle→RGBColor[0,0,1],*)
   PlotLegends → {"c12: (f1,f1,1/2 (f1-f1^2-f1 Sqrt[-3 - 2 f1 + f1^2])) , f1>3"}];
c21 = ParametricPlot3D[{f1, f1, 1/2 (f1 - f1^2 + f1 Sqrt[-3 - 2 f1 + f1^2])},
   {f1, -2, -1}, (*PlotStyle→RGBColor[1,1,0],*)
   PlotLegends → {"c21: (f1,f1,1/2 (f1-f1^2+f1 Sqrt[-3 - 2 f1 + f1^2])) , f1<-1"}];
c22 = ParametricPlot3D[
   {f1, f1, 1/2 (f1 - f1^2 + f1 Sqrt[-3 - 2 f1 + f1^2])},
   {f1, 3, 4}, (*PlotStyle→RGBColor[0,1,1],*)
   PlotLegends → {"c22: (f1,f1,1/2 (f1-f1^2+f1 Sqrt[-3 - 2 f1 + f1^2])) , f1>3"}];
c3 = ParametricPlot3D[{(f3 - Sqrt[-3 f3^2 - 4 f3^3])/(2 (1 + f3)), (f3 + Sqrt[-3 f3^2 - 4 f3^3])/(2 (1 + f3)), f3},
   {f3, -s, -1.5}, PlotLegends → {"c3: one of the two guessing curve"}];


c4 = ParametricPlot3D[{(f3 + Sqrt[-3 f3^2 - 4 f3^3])/(2 (1 + f3)), (f3 - Sqrt[-3 f3^2 - 4 f3^3])/(2 (1 + f3)), f3},
   {f3, -s, -1.5}, PlotLegends → {"c4: one of the two guessing curve"}];

o1 = ParametricPlot3D[{{t, -1, -1}, {-1, t, -1}, {t, 3, -3}, {3, t, -3}}, {t, -s, s},
   PlotStyle → RGBColor[1, 0, 0], PlotLegends → {"o1: four straight line"}];
```

```
o2 = ParametricPlot3D[{f1, f1, -f1 - 2 √2 f1 + f1^2},
    {f1, -1, 3}, PlotStyle → RGBColor[0, 0, 0],
    PlotLegends → {"o2: one of contour intersection with f1=f2 plane"}];
o3 = ParametricPlot3D[{f1, f1, -f1 + 2 √2 f1 + f1^2},
    {f1, -3, 1}, PlotStyle → RGBColor[0, 0, 0],
    PlotLegends → {"o3: one of contour intersection with f1=f2 plane"}];
Show[{surface, c11, c12, c21, c22, c3, c4, o1, o2, o3},
 PlotRange → Automatic, PlotLegends → Automatic]
Show[{surface, c11, c12, c21, c22, c3, c4,
  Graphics3D[{PointSize[0.03], Point[{{1/2 (2 - 3 √2), 1/2 (2 - 3 √2), -3/2 + 1/2 × √2},
      {1/2 (2 + 3 √2), 1/2 (2 + 3 √2), -3/2 - 1/2 × √2}}]}]}]
Show[{surface, c11, (*c12,c21,*)c22, o2, o3,
  Graphics3D[{PointSize[0.03], Point[{{1/2 (2 - 3 √2), 1/2 (2 - 3 √2), -3/2 + 1/2 × √2},
      {1/2 (2 + 3 √2), 1/2 (2 + 3 √2), -3/2 - 1/2 × √2}}]}]},
 PlotRange → Automatic, PlotLegends → Automatic]
(* g denote the surface, c* denote the desired curve,
o* denote some interesting curves also satisfy the contour *)
(* Note that c11 and c12 are actually same function in different input range,
to prevent image output. Same works for c21 and c22 *)
```

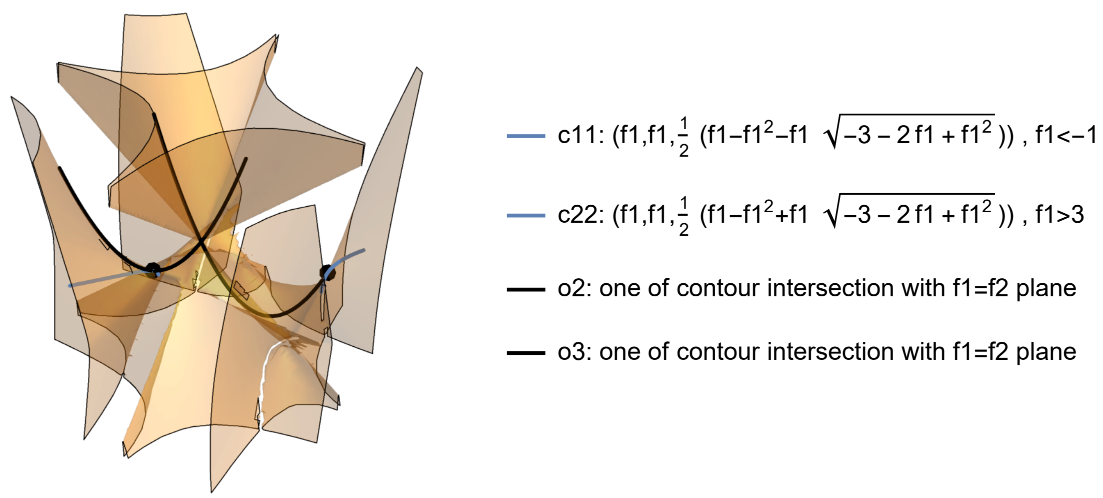
Out[ ]=



Out[ ]=



Out[ ]=



In[ ]:=

```
(* This is used for find the MP of NL *)
Solve[1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2]) - (-f1 + 2 Sqrt[2] f1 + f1^2) == 0, f1]
Solve[1/2 (f1 - f1^2 + f1 Sqrt[-3 - 2 f1 + f1^2]) - (-f1 - 2 Sqrt[2] f1 + f1^2) == 0, f1]
(* This is for find the eigenvalue and eigenvectors of MP and NL *)
Eigenvalues[F[1/2 (2 - 3 Sqrt[2]), 1/2 (2 - 3 Sqrt[2]), -3/2 + 1/2 Sqrt[2]]]
Eigenvectors[F[1/2 (2 - 3 Sqrt[2]), 1/2 (2 - 3 Sqrt[2]), -3/2 + 1/2 Sqrt[2]]]

ListPlot[
 Transpose[Table[Re[Eigenvalues[F[f1, f1, 0.5 (f1 - f1^2 - f1 Sqrt[-3.0 - 2.0 f1 + f1^2])]]],
   {f1, -1.4, -1, 0.0001}]], DataRange → {-1.4, -1}, AxesLabel → {"f1", "Re"},
```

```
 PlotLegends → {"Eigenvalue1", "Eigenvalues2", "Eigenvalues3"},
 PlotStyle → RGBColor[0, 0, 0]]
(*For[i=1,i≤3,i++,Print[ListPlot[
   Transpose[Table[Re[Eigenvalues[F[f1, f1, 0.5 (f1-f1^2-f1 Sqrt[-3.0-2.0 f1+f1^2])]]],
     {f1,-1.4,-1,0.0001}]][[i]],DataRange→{-1.4,-1},
   AxesLabel→{"f1","Re"},PlotLegends→{StringForm["Eigenvalues``",i]},
   PlotStyle→{RGBColor[1,0,0],RGBColor[0,1,0],RGBColor[0,0,1]}[[i]]]]]
 ListPlot[
  Transpose[Table[Im[Eigenvalues[F[f1, f1, 0.5 (f1-f1^2-f1 Sqrt[-3.0-2.0 f1+f1^2])]]],
    {f1,-1.4,-1,0.0001}]],DataRange→{-1.4,-1},AxesLabel→{"f1","Im"},
  PlotLegends→{"Eigenvalue1","Eigenvalues2","Eigenvalues3"},
  PlotStyle→{RGBColor[1,0,0],RGBColor[0,1,0],RGBColor[0,0,1]}]*)
For[i = 1, i ≤ 3, i++, Print[ListPlot[
   Transpose[Table[Im[Eigenvalues[F[f1, f1, 0.5 (f1 - f1^2 - f1 Sqrt[-3.0 - 2.0 f1 + f1^2])]]],
     {f1, -1.4, -1, 0.0001}]][[i]], DataRange → {-1.4, -1},
   AxesLabel → {"f1", "Im"}, PlotLegends → {StringForm["Eigenvalues``", i]},
   PlotStyle → {RGBColor[1, 0, 0], RGBColor[0, 1, 0], RGBColor[0, 0, 1]}[[i]]]]]
(*Refine[Eigenvectors[F[f1,f1,1/2 (f1-f1^2-f1 Sqrt[-3-2 f1+f1^2])]],f1<1]*)
Eigenvalues[F[1/2 (2 + 3 Sqrt[2]), 1/2 (2 + 3 Sqrt[2]), -3/2 - 1/2 Sqrt[2]]]
Eigenvectors[F[1/2 (2 + 3 Sqrt[2]), 1/2 (2 + 3 Sqrt[2]), -3/2 - 1/2 Sqrt[2]]]
ListLinePlot[
 Transpose[Table[Re[Eigenvalues[F[f1, f1, 1/2 (f1 - f1^2 + f1 Sqrt[-3 - 2 f1 + f1^2])]]],
   {f1, 3, 3.4, 0.0001}]], DataRange → {3, 3.4}, AxesLabel → {"f1", "Re"},
 PlotLegends → {"Eigenvalue1", "Eigenvalues2", "Eigenvalues3"},
 PlotStyle → RGBColor[0, 0, 0]]
ListLinePlot[
 Transpose[Table[Im[Eigenvalues[F[f1, f1, 1/2 (f1 - f1^2 + f1 Sqrt[-3 - 2 f1 + f1^2])]]],
   {f1, 3, 3.4, 0.0001}]], DataRange → {3, 3.4}, AxesLabel → {"f1", "Im"},
 PlotLegends → {"Eigenvalue1", "Eigenvalues2", "Eigenvalues3"}]
```

Out[ ]=

$$\left\{\{f1 \to 0\}, \left\{f1 \to \frac{1}{2}\left(2 - 3\sqrt{2}\right)\right\}\right\}$$

Out[ ]=

$$\left\{\{f1 \to 0\}, \left\{f1 \to \frac{1}{2}\left(2+3\sqrt{2}\right)\right\}\right\}$$

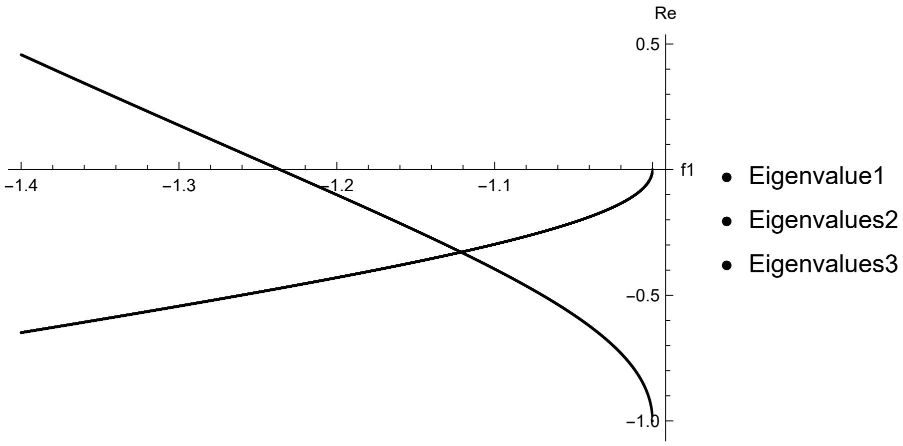
Out[ ]=

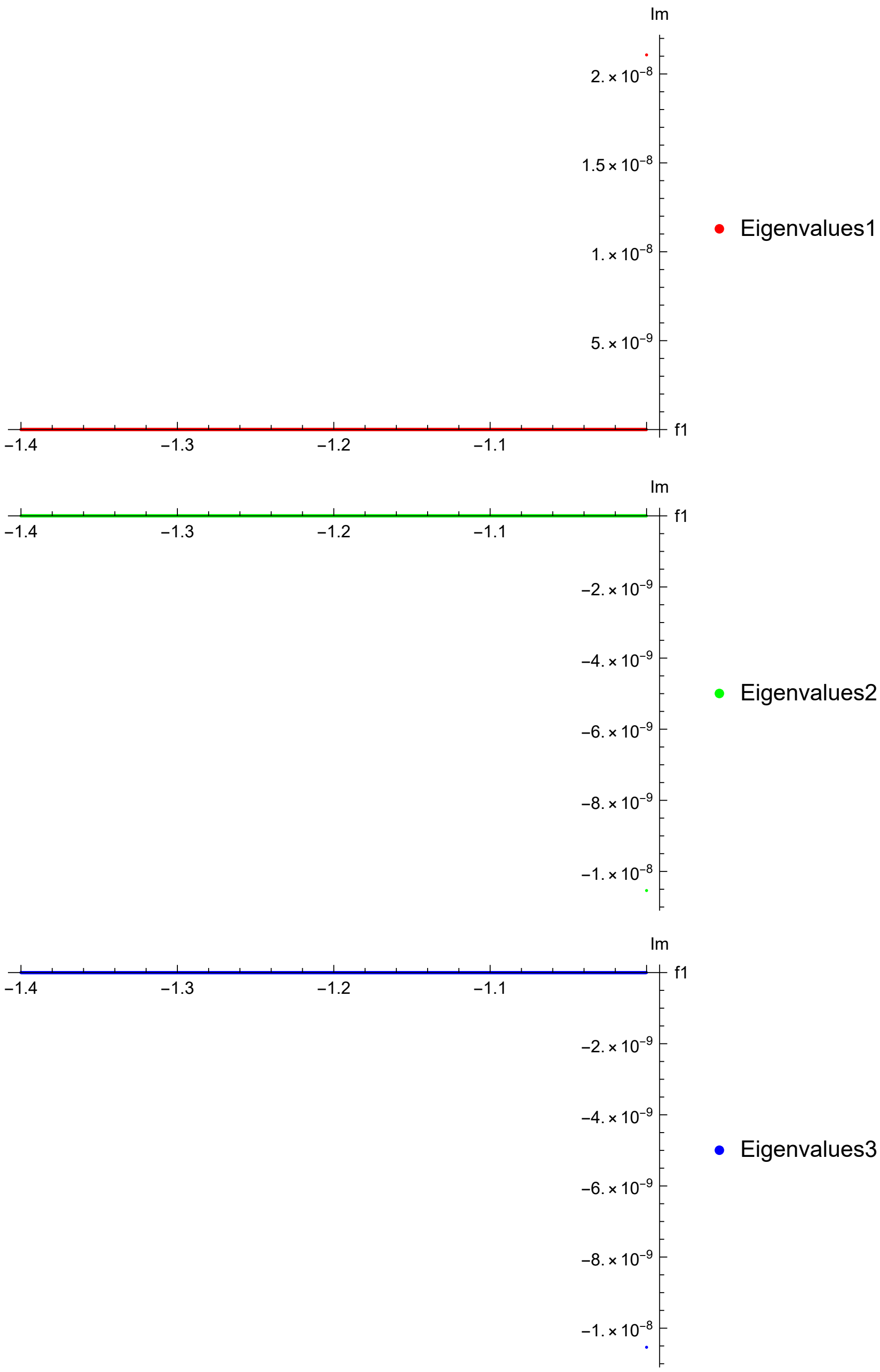
$$\left\{\frac{1}{2}\left(5-4\sqrt{2}\right), \frac{1}{2}\left(5-4\sqrt{2}\right), \frac{1}{2}\left(5-4\sqrt{2}\right)\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{-2+3\sqrt{2}}{2\left(-3+\sqrt{2}\right)}, 0, 1\right\}, \left\{-\frac{-2+3\sqrt{2}}{2\left(-3+\sqrt{2}\right)}, 1, 0\right\}, \{0, 0, 0\}\right\}$$

Out[ ]=





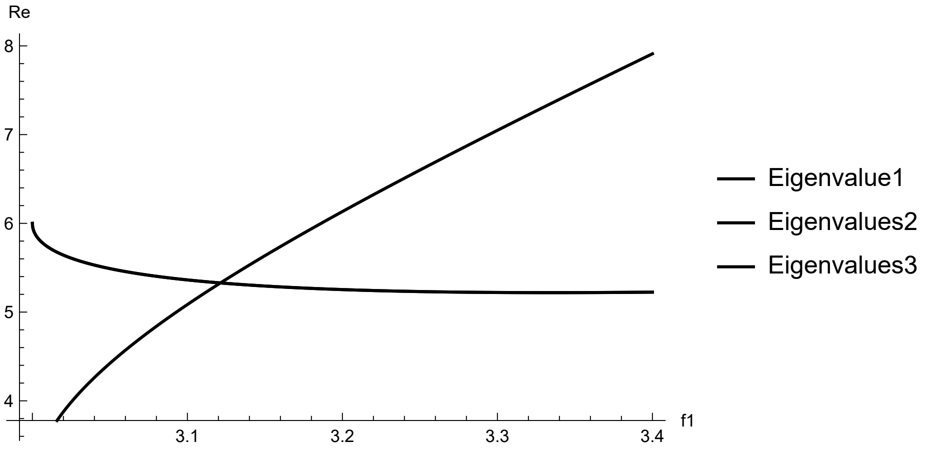
Out[ ]=

$$\left\{\frac{1}{2}\left(5+4\sqrt{2}\right), \frac{1}{2}\left(5+4\sqrt{2}\right), \frac{1}{2}\left(5+4\sqrt{2}\right)\right\}$$

Out[ ]=

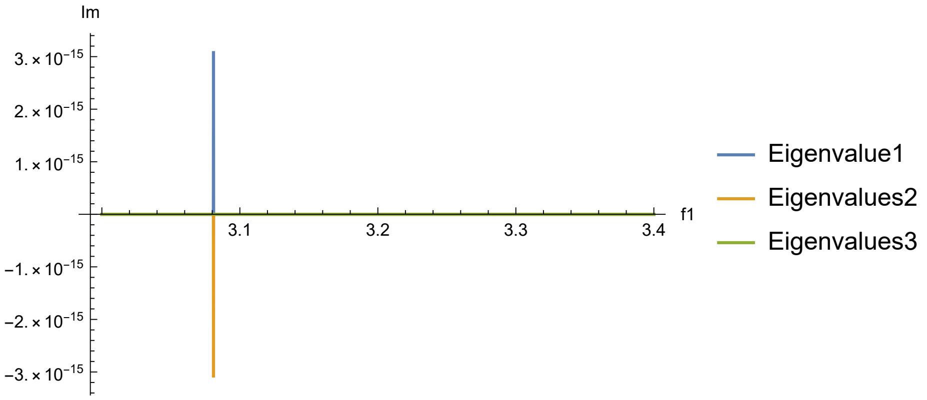
$$\left\{\left\{-\frac{2+3\sqrt{2}}{2\left(3+\sqrt{2}\right)}, 0, 1\right\}, \left\{-\frac{2+3\sqrt{2}}{2\left(3+\sqrt{2}\right)}, 1, 0\right\}, \{0, 0, 0\}\right\}$$

Out[ ]=



Re
8
7
6
5
4
3.1
3.2
3.3
3.4
f1
Eigenvalue1
Eigenvalues2
Eigenvalues3

Out[ ]=



Im
3. × 10^-15
2. × 10^-15
1. × 10^-15
−1. × 10^-15
−2. × 10^-15
−3. × 10^-15
3.1
3.2
3.3
3.4
f1
Eigenvalue1
Eigenvalues2
Eigenvalues3

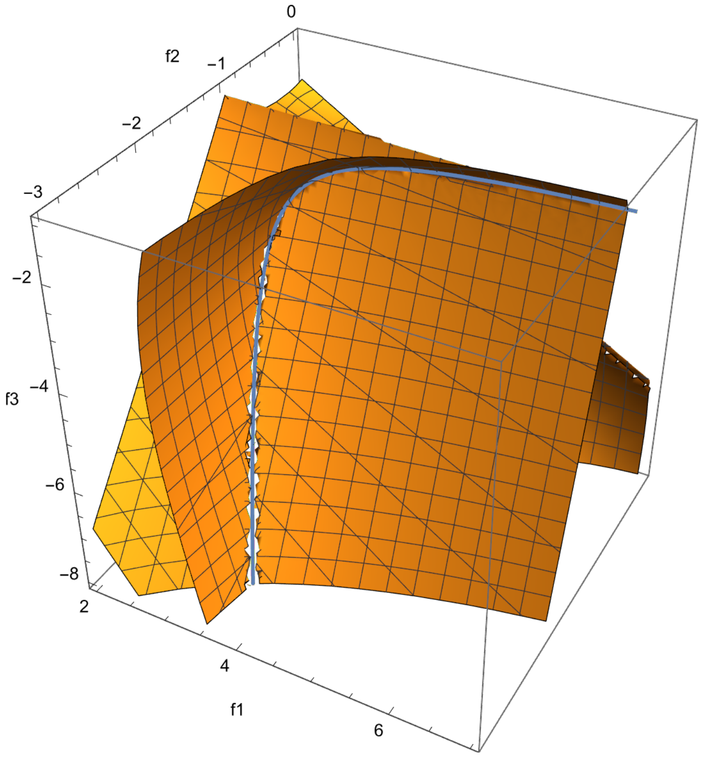
In[ ]:=

```
(* This is for the third and forth MP (if they exist) *)
surface1 =
  ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
   {f1, 2, 7}, {f2, -3, 0}, {f3, -8, -1}, AxesLabel → Automatic];
```

$$c = \text{ParametricPlot3D}\left[\left\{\frac{f3 - \sqrt{-3\,f3^2 - 4\,f3^3}}{2\,(1 + f3)}, \frac{f3 + \sqrt{-3\,f3^2 - 4\,f3^3}}{2\,(1 + f3)}, f3\right\}, \{f3, -8, -1\}\right];$$

```
Show[{surface1, c}]
```

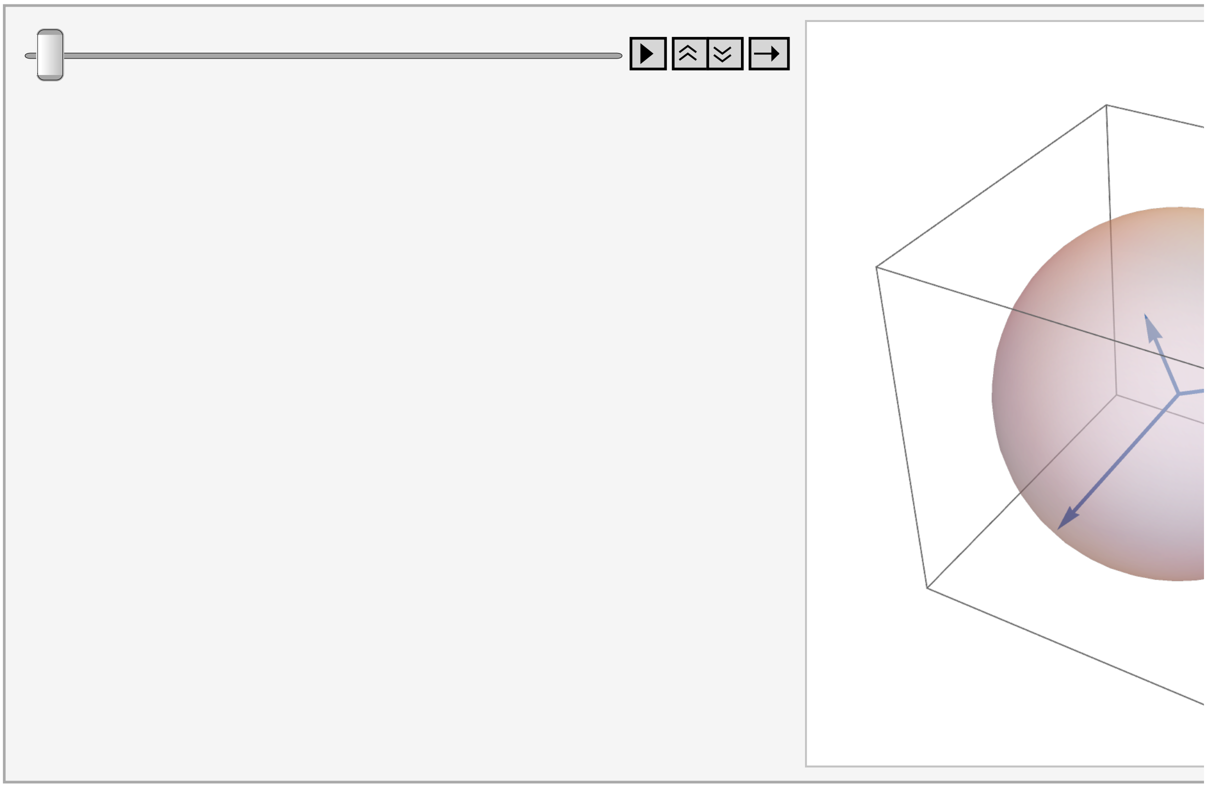
Out[ ]=



In[ ]:=

In[ ]:=
```
(* This is for visualizing the eigenvectors alone NL. One *)
G1 = Graphics3D[{Opacity[0.4], Ball[]}];
ListAnimate[Table[Show[{G1, ParametricPlot3D[
    {Normalize[Eigenvectors[F[f1, f1, 1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2])]][[1]]],
      Normalize[Eigenvectors[F[f1, f1, 1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2])]][[2]]],
      Normalize[Eigenvectors[F[f1, f1, 1/2 (f1 - f1^2 - f1 Sqrt[-3 - 2 f1 + f1^2])]][[3]]]} *
     u, {u, 0, 1}] /. Line → Arrow}],
  {f1, -2.5, 1/2 (2 - 3 Sqrt[2]), 0.001}], AnimationRunning → False]
```

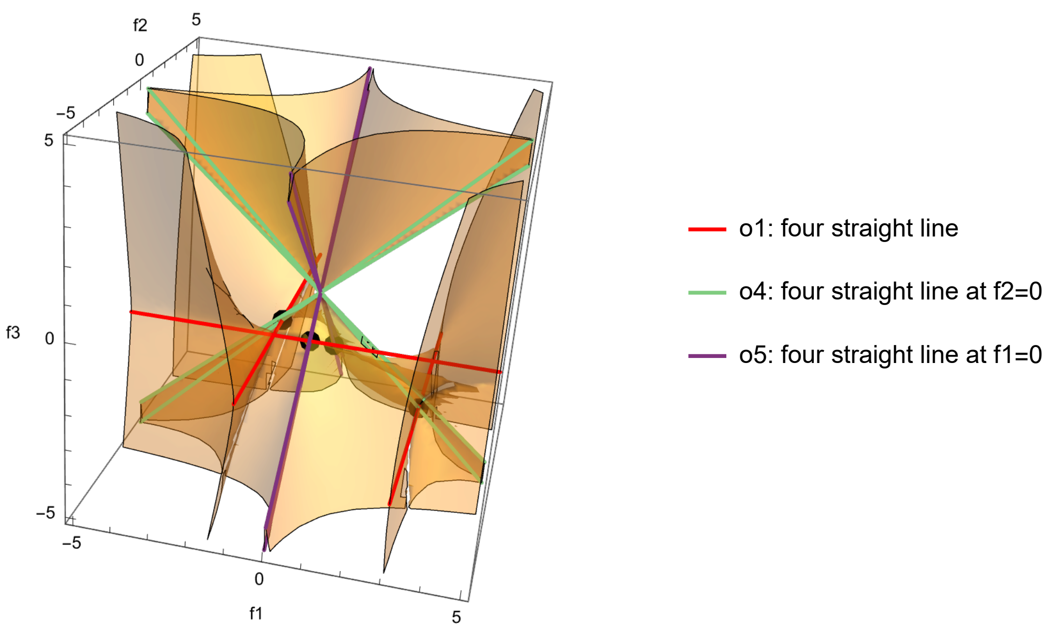
Out[ ]=



```
(* Cone point: local property *)
```

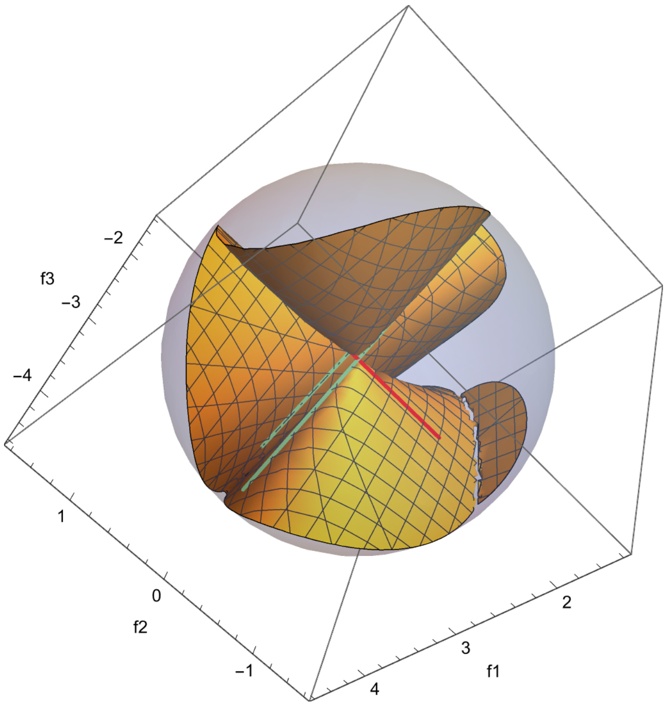
In[ ]:=
```
ClearAll["Global`*"]
gradedForm /: MakeBoxes[gradedForm[poly_], form_] :=
  Module[{t}, With[{vars = Alternatives @@ Variables@poly},
    RowBox[Riffle[RowBox[{"(", MakeBoxes[#, form], ")"}] & /@
       CoefficientList[poly /. v : vars :→ t * v, t], "+"]]]];
F[f1_, f2_, f3_] := ( f1 f2 f1 f2
                      -f1  f1 f3
                      -f2  f3 f2 )
g[f1_, f2_, f3_] :=
  Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] // gradedForm;
s = 5;
surface =
  ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
   {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, Mesh → None,
   ContourStyle → Opacity[0.4], AxesLabel → Automatic];
o1 = ParametricPlot3D[{{t, -1, -1}, {-1, t, -1}, {t, 3, -3}, {3, t, -3}}, {t, -s, s},
   PlotStyle → RGBColor[1, 0, 0], PlotLegends → {"o1: four straight line"}];
o4 = ParametricPlot3D[{{t, 0, -t}, {t, 0, t}, {t, 0, √3 / 2 t}, {t, 0, -√3 / 2 t}},
   {t, -s, s}, PlotStyle → RGBColor[.5, .8, .5],
   PlotLegends → {"o4: four straight line at f2=0"}];
o5 = ParametricPlot3D[{{0, t, -t}, {0, t, t}, {0, t, √3 / 2 t}, {0, t, -√3 / 2 t}},
   {t, -s, s}, PlotStyle → RGBColor[.5, .2, .5],
   PlotLegends → {"o5: four straight line at f1=0"}];
Show[{surface, o1, o4, o5, Graphics3D[
   {PointSize[0.03], Point[{{3, 0, -3}, {0, 3, -3}, {-1, 0, -1}, {0, -1, -1}}]}]},
 PlotRange → Automatic, PlotLegends → Automatic]
```

Out[ ]=

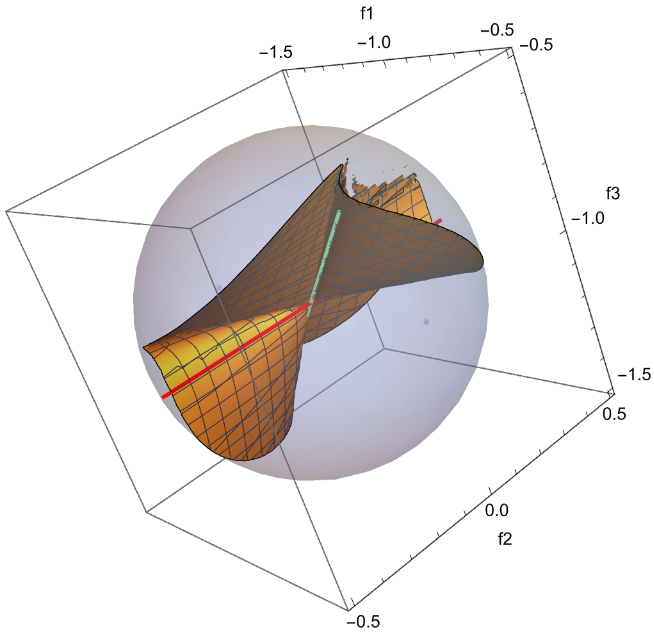


In[ ]:=
```
neighborhoodCone1 =
  ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
   {f1, f2, f3} ∈ Ball[{3, 0, -3}, 1.6]];
coneLine11 = ParametricPlot3D[{3, t, -3}, {t, -1, 1}, PlotStyle → RGBColor[1, 0, 0]];
coneLine12 = ParametricPlot3D[{t, 0, -t}, {t, 2, 4}, PlotStyle → RGBColor[.5, .8, .5]];
coneLine13 = ParametricPlot3D[{t, -1, -1}, {t, 0, 6}, PlotStyle → RGBColor[1, 0, 0]];
coneLine14 =
  ParametricPlot3D[{t, 0, -(√3/2) t}, {t, 2, 4}, PlotStyle → RGBColor[.5, .8, .5]];
selfIntersection11 =
  ParametricPlot3D[{(f3 - √(-3 f3^2 - 4 f3^3))/(2 (1 + f3)), (f3 + √(-3 f3^2 - 4 f3^3))/(2 (1 + f3)), f3}, {f3, -5, -1.5}];
neighborhoodCone2 =
  ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
   {f1, f2, f3} ∈ Ball[{-1, 0, -1}, 0.5]];


coneLine21 = ParametricPlot3D[{-1, t, -1}, {t, -1, 1}, PlotStyle → RGBColor[1, 0, 0]];
coneLine22 =
  ParametricPlot3D[{-t, 0, -t}, {t, .7, 1.3}, PlotStyle → RGBColor[.5, .8, .5]];
selfIntersection21 = ParametricPlot3D[{f1, f1, 1/2 (f1 - f1^2 + f1 √(-3 - 2 f1 + f1^2))},
   {f1, -2, -1}, PlotStyle → RGBColor[1, 1, 0]];
selfIntersection22 = ParametricPlot3D[{f1, f1, 1/2 (f1 - f1^2 - f1 √(-3 - 2 f1 + f1^2))},
   {f1, -2, -1}, PlotStyle → RGBColor[0, 1, 0]];

Show[{neighborhoodCone1, coneLine11, coneLine12, coneLine14(* ,selfIntersection11,
  coneLine13 *)}, AxesLabel → {f1, f2, f3}, PlotRange → Automatic]
Show[{neighborhoodCone2, coneLine21, coneLine22(* ,
  selfIntersection21,selfIntersection22 *)}, AxesLabel → {f1, f2, f3}]
```

Out[ ]=



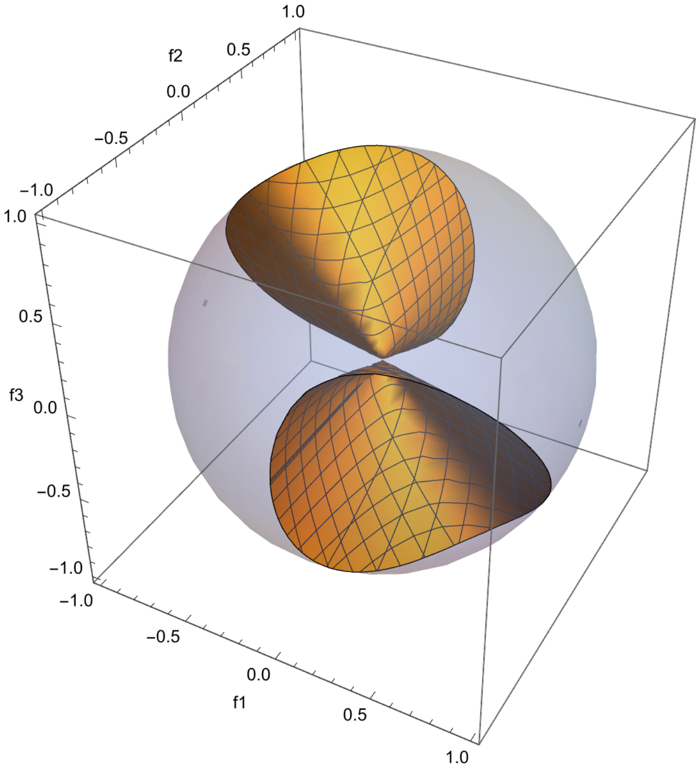
Out[ ]=



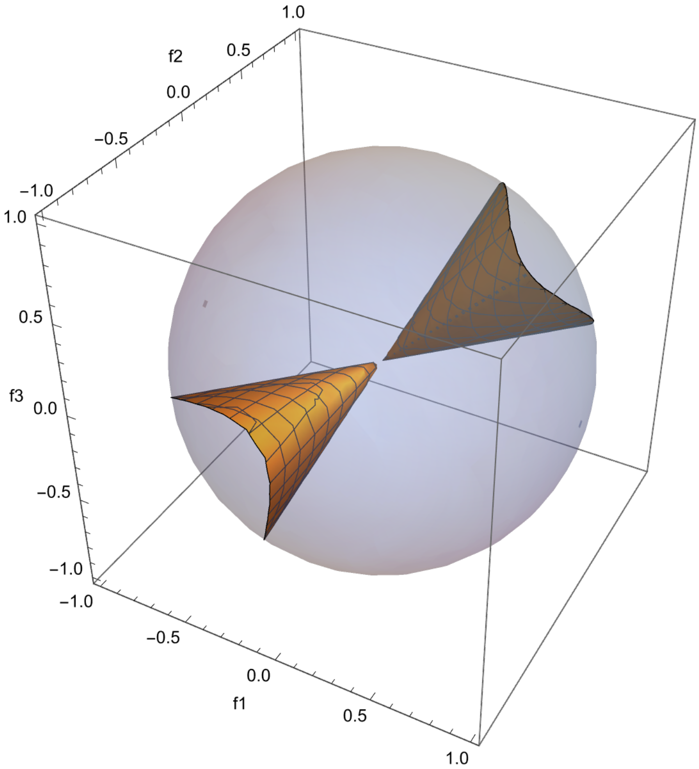
In[ ]:=

```
(* This is for the local approximation of
 sw4c4 at the cone point (3,0,-3) and (-1,0,-1) *)
Simplify[g[f1 + 3, f2, f3 - 3]];
Simplify[g[f1 - 1, f2, f3 - 1]];
ContourPlot3D[324 f1^2 + 972 f1 f2 + 648 f1 f3 + 324 f3^2 == 0,
 {f1, f2, f3} ∈ Ball[{0, 0, 0}, 1], AxesLabel → {f1, f2, f3}]
ContourPlot3D[4 f1^2 - 4 f1 f2 - 8 f1 f3 + 4 f3^2 == 0,
 {f1, f2, f3} ∈ Ball[{0, 0, 0}, 1], AxesLabel → {f1, f2, f3}]
```

Out[ ]=



Out[ ]=



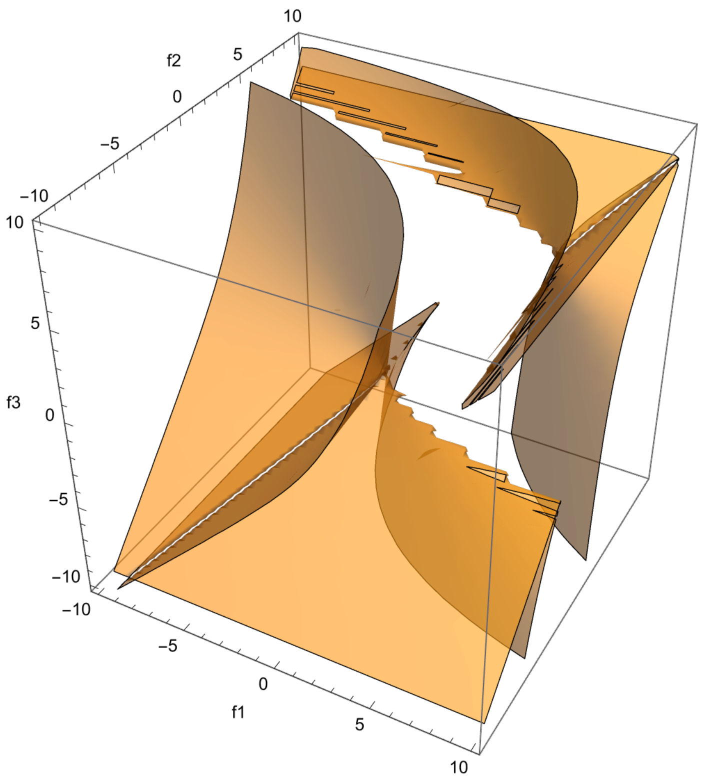
In[∘]:=

```
(* MP of sw2 *)
ClearAll["Global`*"]
F[f1_, f2_, f3_] := ( 1 - f1 - f2   f1        f2
                      -f1           f1 - f3   f3
                      -f2           f3        f2 - f3 )
Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω]
s = 10;
plot1 = ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
  {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, AxesLabel → Automatic,
  Mesh → None, ContourStyle → Opacity[0.6] ]
```

Out[∘]=

$$f1^2 - 4 f1^3 - 2 f1 f2 + 4 f1^2 f2 + 12 f1^3 f2 + f2^2 + 4 f1 f2^2 - 20 f1^2 f2^2 - 12 f1^3 f2^2 - 4 f2^3 + 12 f1 f2^3 - 12 f1^2 f2^3 + 4 f1^3 f2^3 + 4 f1^2 f3 - 12 f1^3 f3 - 24 f1^2 f2 f3 + 24 f1^3 f2 f3 + 4 f2^2 f3 - 24 f1 f2^2 f3 + 104 f1^2 f2^2 f3 - 12 f1^3 f2^2 f3 - 12 f2^3 f3 + 24 f1 f2^3 f3 - 12 f1^2 f2^3 f3 + 4 f3^2 - 24 f1 f3^2 + 36 f1^2 f3^2 - 12 f1^3 f3^2 - 24 f2 f3^2 + 128 f1 f2 f3^2 - 156 f1^2 f2 f3^2 + 12 f1^3 f2 f3^2 + 36 f2^2 f3^2 - 156 f1 f2^2 f3^2 + 28 f1^2 f2^2 f3^2 - 12 f2^3 f3^2 + 12 f1 f2^3 f3^2 + 16 f3^3 - 72 f1 f3^3 + 64 f1^2 f3^3 - 4 f1^3 f3^3 - 72 f2 f3^3 + 176 f1 f2 f3^3 - 20 f1^2 f2 f3^3 + 64 f2^2 f3^3 - 20 f1 f2^2 f3^3 - 4 f2^3 f3^3 + 16 f3^4 - 48 f1 f3^4 + 4 f1^2 f3^4 - 48 f2 f3^4 + 8 f1 f2 f3^4 + 4 f2^2 f3^4$$

Out[∘]=



In[∘]:=

```
Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] /. {f1 → 0, f2 → 0, f3 → -1/2}
```

Out[∘]=

```
0
```

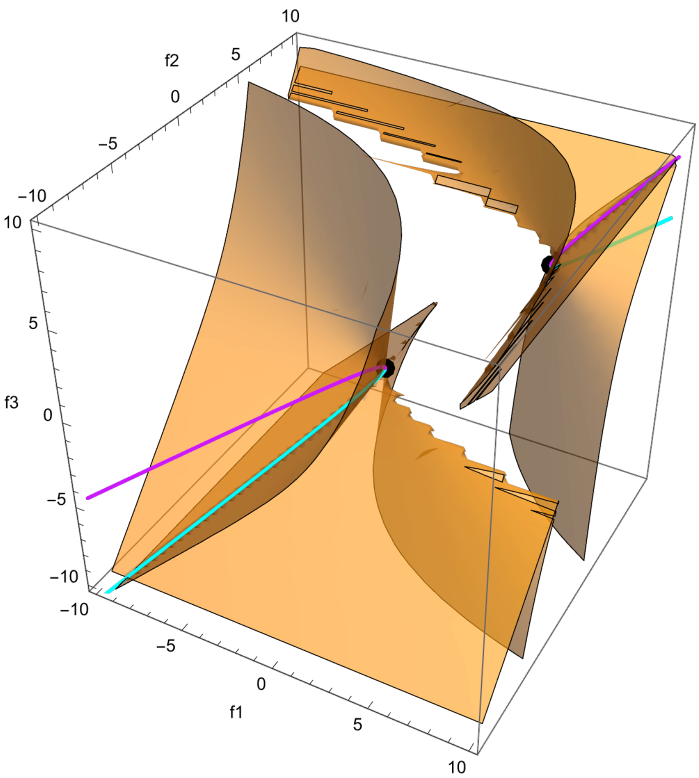
In[∘]:=

In[ ]:=
```
h1[f1_, f2_, f3_] := Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] ;
h2[f1_, f2_, f3_] := f1 - f2;
Solve[h1[f1, f2, f3] == 0 && h2[f1, f2, f3] == 0, {f2, f3}]
plot2 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 - Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0, 1, 1]];
plot3 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 + Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0.8, 0.1, 1]];
plot4 = Graphics3D[{PointSize[0.03],
    Point[{{3 + 2 Sqrt[2], 3 + 2 Sqrt[2], 2 + 3/2 Sqrt[2]}, {3 - 2 Sqrt[2], 3 - 2 Sqrt[2], 2 - 3/2 Sqrt[2]}}]}];
plot5 = Graphics3D[{PointSize[0.03], Red, Point[{3 + 2 Sqrt[2], 3 + 2 Sqrt[2], 2 + 3/2 Sqrt[2]}]}];
plot6 = Graphics3D[{PointSize[0.03], Blue, Point[{3 - 2 Sqrt[2], 3 - 2 Sqrt[2], 2 - 3/2 Sqrt[2]}]}];
Show[{plot1, plot2, plot3, plot4}]
Show[{plot1, plot2, plot3, plot5, plot6}]
```

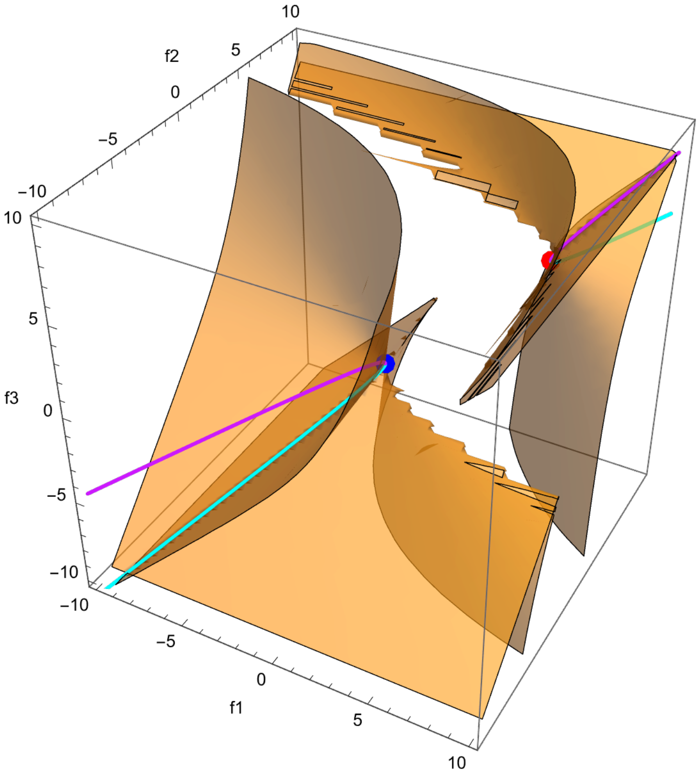
Out[ ]=

$$\left\{\left\{f2 \to f1,\ f3 \to \frac{1}{4}\left(-1 + 3\,f1 - \sqrt{1 - 6\,f1 + f1^2}\right)\right\},\ \left\{f2 \to f1,\ f3 \to \frac{1}{4}\left(-1 + 3\,f1 + \sqrt{1 - 6\,f1 + f1^2}\right)\right\}\right\}$$

Out[ ]=



Out[ ]=



In[ ]:=

```
Eigenvalues[F[3 + 2 √2, 3 + 2 √2, 2 + 3/2 √2]]
Eigenvectors[F[3 + 2 √2, 3 + 2 √2, 2 + 3/2 √2]]
Eigenvalues[F[3 - 2 √2, 3 - 2 √2, 2 - 3/2 √2]]
Eigenvectors[F[3 - 2 √2, 3 - 2 √2, 2 - 3/2 √2]]
```

Out[ ]=

$$\left\{-1-\sqrt{2},\ -1-\sqrt{2},\ -1-\sqrt{2}\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{-3-2\sqrt{2}}{4+3\sqrt{2}},\ 0,\ 1\right\},\ \left\{-\frac{-3-2\sqrt{2}}{4+3\sqrt{2}},\ 1,\ 0\right\},\ \{0,\ 0,\ 0\}\right\}$$

Out[ ]=

$$\left\{-1+\sqrt{2},\ -1+\sqrt{2},\ -1+\sqrt{2}\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{3-2\sqrt{2}}{-4+3\sqrt{2}},\ 0,\ 1\right\},\ \left\{-\frac{3-2\sqrt{2}}{-4+3\sqrt{2}},\ 1,\ 0\right\},\ \{0,\ 0,\ 0\}\right\}$$

In[ ]:=

```
Eigenvalues[F[f1, f1, 1/4 (-1 + 3 f1 - Sqrt[1 - 6 f1 + f1^2])]]
Eigenvectors[F[f1, f1, 1/4 (-1 + 3 f1 - Sqrt[1 - 6 f1 + f1^2])]]
Plot[{1/2 (1 - f1 - Sqrt[1 - 6 f1 + f1^2]), 1/2 (1 - f1 + Sqrt[1 - 6 f1 + f1^2])},
 {f1, -1, 3 - 2 Sqrt[2]}, PlotLegends -> "Expressions"]
Plot[{1/2 (1 - f1 - Sqrt[1 - 6 f1 + f1^2]), 1/2 (1 - f1 + Sqrt[1 - 6 f1 + f1^2])},
 {f1, 3 + 2 Sqrt[2], 7}, PlotLegends -> "Expressions"]
```

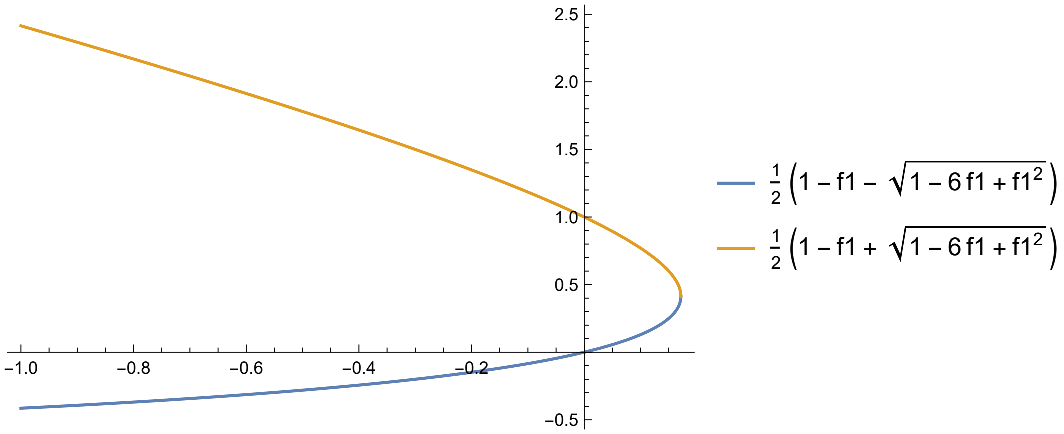
Out[ ]=

$$\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right),\ \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right),\ \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}$$

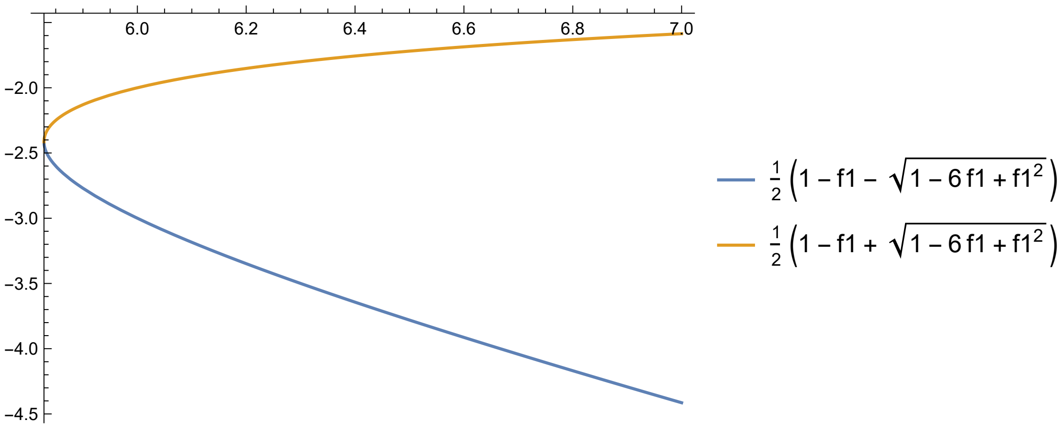
Out[ ]=

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1},\ 0,\ 1\right\},\ \left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1},\ 1,\ 0\right\},\ \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1},\ 1,\ 1\right\}\right\}$$

Out[ ]=



Out[ ]=



In[ ]:=

```
G1 = Graphics3D[{Opacity[0.4], Ball[]}];
```

$$G2 = \text{ParametricPlot3D}\left[\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\Big\},$$

$$\left\{f1,\ 0.1,\ 3-2\sqrt{2}\right\},\ \texttt{PlotStyle}\rightarrow\texttt{RGBColor[1, 1, 0]}\Big];$$

$$\texttt{G3 = ParametricPlot3D}\Big[$$

$$\Big\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\Big\},$$

$$\left\{f1,\ 3-2\sqrt{2},\ 0.1\right\},\ \texttt{PlotStyle}\rightarrow\texttt{RGBColor[1, 1, 0]}\Big];$$

$$\texttt{Animate}\Big[\texttt{Show}\Big[$$

$$\Big\{\texttt{G1, G2, ParametricPlot3D}\Big[\Big\{\Big\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\Big\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},$$

$$\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\Big\}\Big\}*u,\ \{u,\ 0,\ 1\}\Big]\ \texttt{/. Line}\rightarrow\texttt{Arrow}\Big\}\Big],$$

$$\left\{f1,\ 0.1,\ 3-2\sqrt{2}\right\},\ \texttt{AnimationRunning}\rightarrow\texttt{False}\Big]$$

$$\texttt{Animate}\Big[\texttt{Show}\Big[\Big\{\texttt{G1, G3, ParametricPlot3D}\Big[$$

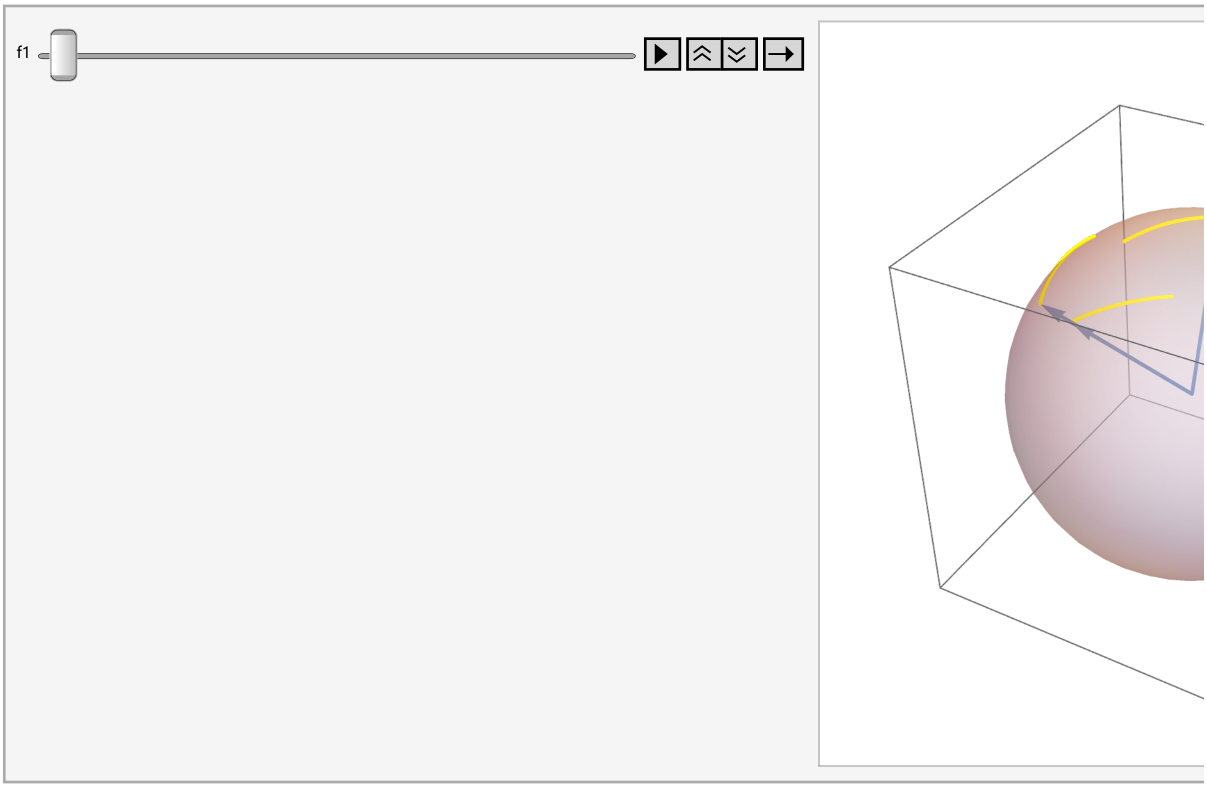
$$\Bigg\{\Bigg\{-\frac{1-3\,\mathrm{f1}+\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}}{2\,\mathrm{f1}\sqrt{2+\frac{\left(1-3\,\mathrm{f1}+\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{4\,\mathrm{f1}^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,\mathrm{f1}+\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{4\,\mathrm{f1}^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,\mathrm{f1}+\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{4\,\mathrm{f1}^2}}}\Bigg\},$$

$$\Bigg\{-\frac{1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}}{4\,\mathrm{f1}\sqrt{1+\frac{\left(1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{16\,\mathrm{f1}^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{16\,\mathrm{f1}^2}}}\Bigg\},$$

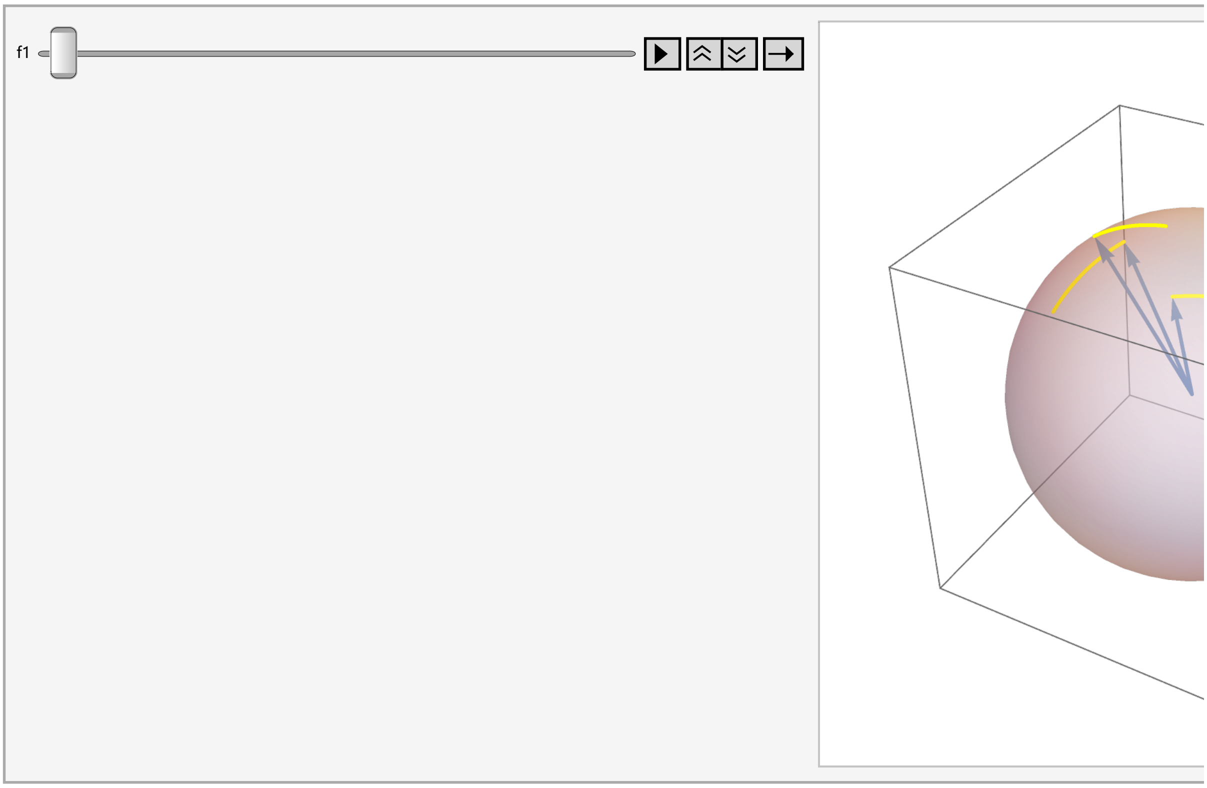
$$\Bigg\{-\frac{1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}}{4\,\mathrm{f1}\sqrt{1+\frac{\left(1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{16\,\mathrm{f1}^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,\mathrm{f1}-\sqrt{1-6\,\mathrm{f1}+\mathrm{f1}^2}\right)^2}{16\,\mathrm{f1}^2}}},\ 0\Bigg\}\Bigg\} * u,\ \{u, 0, 1\}\Big]\ /.$$

$$\mathrm{Line} \to \mathrm{Arrow}\}\Big],\ \left\{\mathrm{f1},\ 3-2\sqrt{2},\ 0.1\right\},\ \mathrm{AnimationRunning} \to \mathrm{False}\Big]$$

Out[ ]=



Out[ ]=



In[ ]:= Eigenvalues[F[f1, f1, $\frac{1}{4}\left(-1+3\,f1+\sqrt{1-6\,f1+f1^2}\right)$]]

Eigenvectors[F[f1, f1, $\frac{1}{4}\left(-1+3\,f1+\sqrt{1-6\,f1+f1^2}\right)$]]

Out[ ]=

$$\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right),\ \frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right),\ \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1},\ 1,\ 1\right\},\ \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1},\ 0,\ 1\right\},\ \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1},\ 1,\ 0\right\}\right\}$$

In[ ]:= Eigenvalues[F[0, 0, $-\frac{1}{2}$]]

Eigenvectors[F[0, 0, $-\frac{1}{2}$]]

Out[ ]=

{1, 1, 0}

Out[ ]=

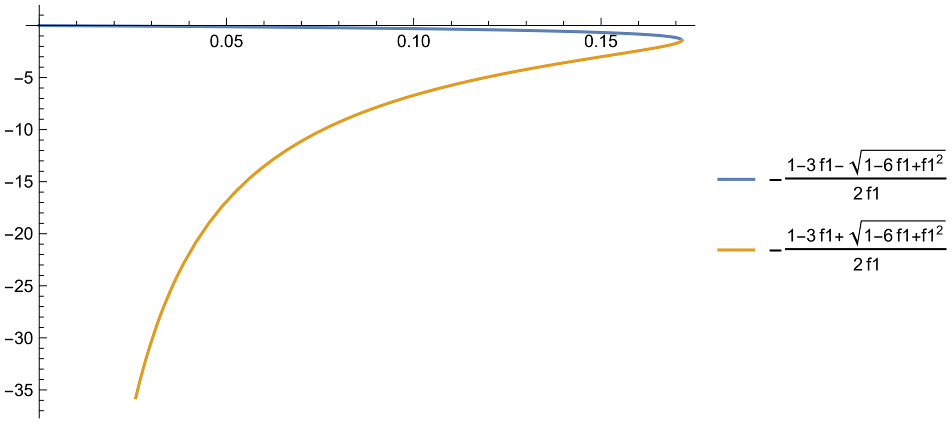
{{0, -1, 1}, {1, 0, 0}, {0, 1, 1}}

In[∘]:=
```
Eigenvalues[F[0, 0, 0]]
Eigenvectors[F[0, 0, 0]]
```
Out[∘]=
```
{1, 0, 0}
```
Out[∘]=
```
{{1, 0, 0}, {0, 0, 1}, {0, 1, 0}}
```
In[∘]:=

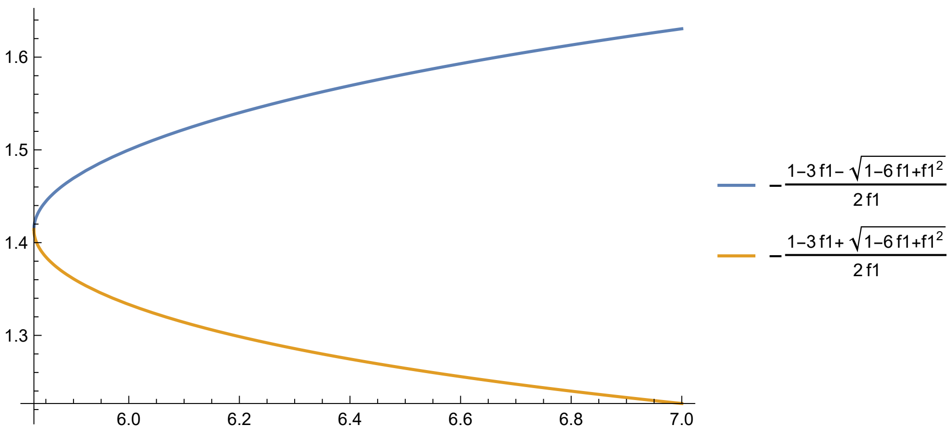
Plot$\left[\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1},\ -\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1}\right\},\right.$ $\left.\left\{f1,\ 0,\ 3-2\sqrt{2}\right\},\ \text{PlotLegends}\to\text{"Expressions"}\right]$

Plot$\left[\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1},\ -\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1}\right\},\right.$ $\left.\left\{f1,\ 3+2\sqrt{2},\ 7\right\},\ \text{PlotLegends}\to\text{"Expressions"}\right]$

Out[∘]=



Out[∘]=



In[∘]:=

$$G4 = \text{ParametricPlot3D}\left[\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},\right.\right.$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\},$$

$\{f1, 20, 3+2\sqrt{2}\}$, PlotStyle → RGBColor[1, 1, 0]];

G5 = ParametricPlot3D[

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},0,\frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left.\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},0\right\}\right\},$$

$\{f1, 3+2\sqrt{2}, 20\}$, PlotStyle → RGBColor[1, 1, 0]];

Animate[Show[

$$\left\{G1, G4, ParametricPlot3D\left[\left\{\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},0,\frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},\right.\right.\right.\right.$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\right.$$

$$\left.\left.\left.\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\right\}*u,\ \{u, 0, 1\}\right]\ /.\ Line \to Arrow\right\}\Big],$$

$\{f1, 20, 3+2\sqrt{2}\}$, AnimationRunning → False]

Animate[Show[{G1, G5, ParametricPlot3D[

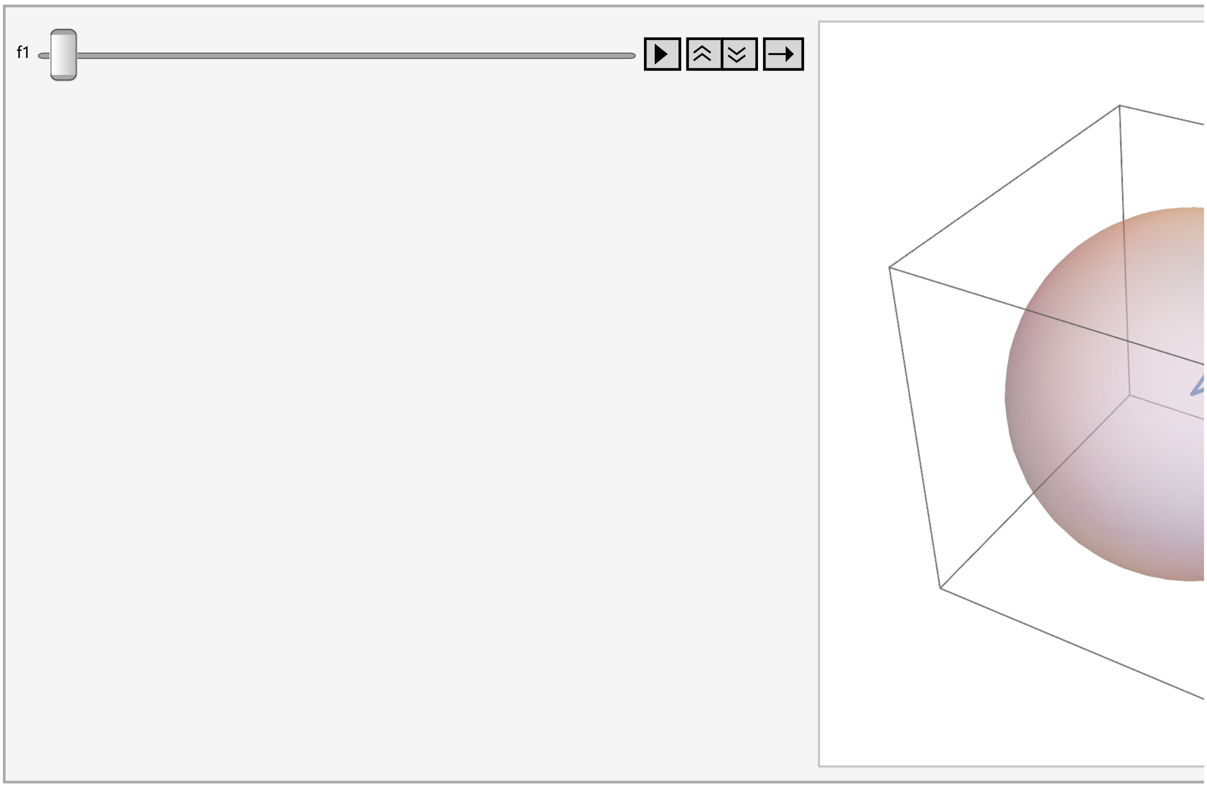
$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

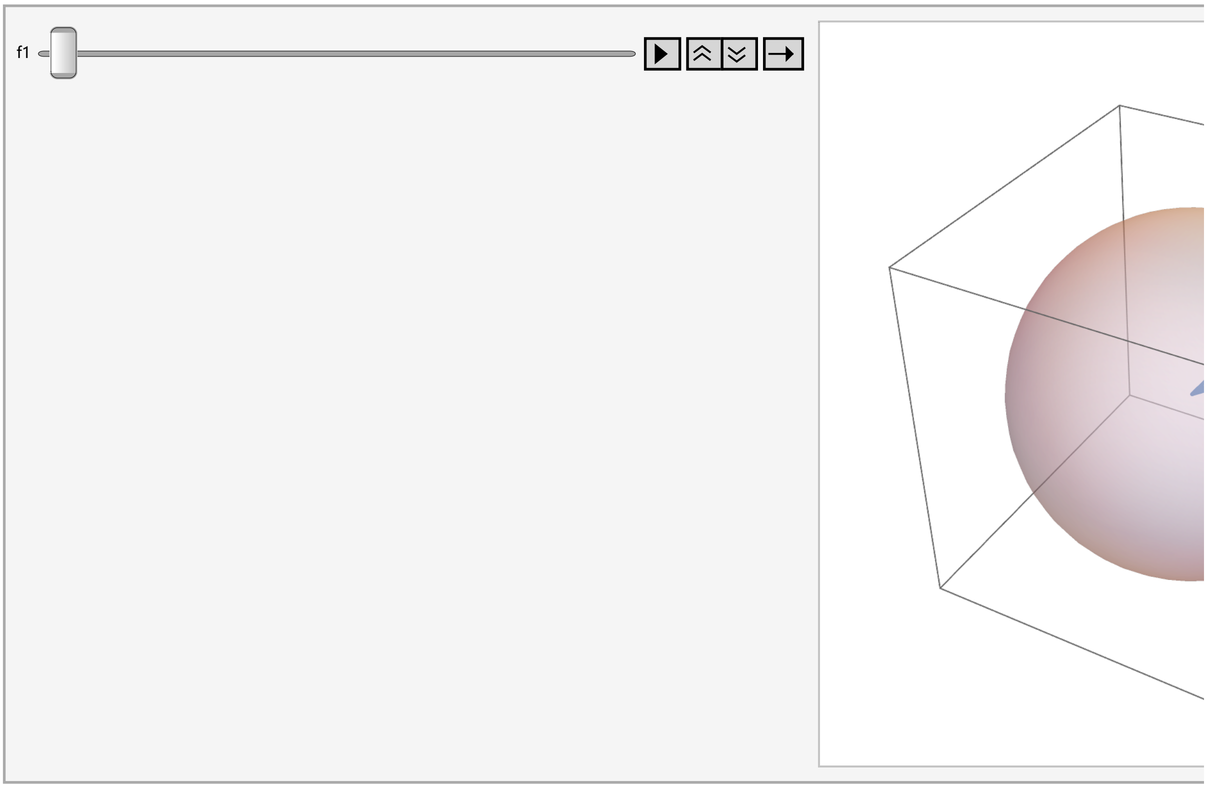
$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\right\} * u,\ \{u,\ 0,\ 1\}\Big]\ /.$$

$$\text{Line} \to \text{Arrow}\}\Big],\ \left\{f1,\ 3+2\sqrt{2},\ 20\right\},\ \text{AnimationRunning} \to \text{False}\Big]$$

Out[ ]=



Out[ ]=

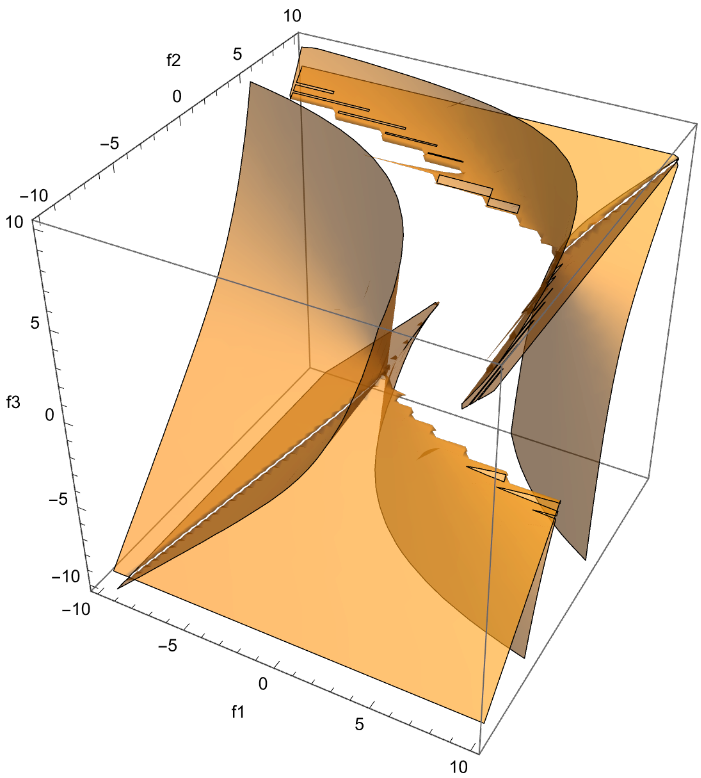


In[ ]:=
```
(* MP of sw2 *)
ClearAll["Global`*"]
F[f1_, f2_, f3_] := {{1 - f1 - f2, f1, f2}, {-f1, f1 - f3, f3}, {-f2, f3, f2 - f3}}
Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω]
s = 10;
plot1 = ContourPlot3D[Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] == 0,
  {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, AxesLabel → Automatic,
  Mesh → None, ContourStyle → Opacity[0.6] ]
```

Out[ ]=

$$f1^2 - 4 f1^3 - 2 f1 f2 + 4 f1^2 f2 + 12 f1^3 f2 + f2^2 + 4 f1 f2^2 - 20 f1^2 f2^2 - 12 f1^3 f2^2 - 4 f2^3 + 12 f1 f2^3 - 12 f1^2 f2^3 + 4 f1^3 f2^3 + 4 f1^2 f3 - 12 f1^3 f3 - 24 f1^2 f2 f3 + 24 f1^3 f2 f3 + 4 f2^2 f3 - 24 f1 f2^2 f3 + 104 f1^2 f2^2 f3 - 12 f1^3 f2^2 f3 - 12 f2^3 f3 + 24 f1 f2^3 f3 - 12 f1^2 f2^3 f3 + 4 f3^2 - 24 f1 f3^2 + 36 f1^2 f3^2 - 12 f1^3 f3^2 - 24 f2 f3^2 + 128 f1 f2 f3^2 - 156 f1^2 f2 f3^2 + 12 f1^3 f2 f3^2 + 36 f2^2 f3^2 - 156 f1 f2^2 f3^2 + 28 f1^2 f2^2 f3^2 - 12 f2^3 f3^2 + 12 f1 f2^3 f3^2 + 16 f3^3 - 72 f1 f3^3 + 64 f1^2 f3^3 - 4 f1^3 f3^3 - 72 f2 f3^3 + 176 f1 f2 f3^3 - 20 f1^2 f2 f3^3 + 64 f2^2 f3^3 - 20 f1 f2^2 f3^3 - 4 f2^3 f3^3 + 16 f3^4 - 48 f1 f3^4 + 4 f1^2 f3^4 - 48 f2 f3^4 + 8 f1 f2 f3^4 + 4 f2^2 f3^4$$

Out[ ]=



In[ ]:=
```
Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] /. {f1 → 0, f2 → 0, f3 → -1/2}
```

Out[ ]=

0

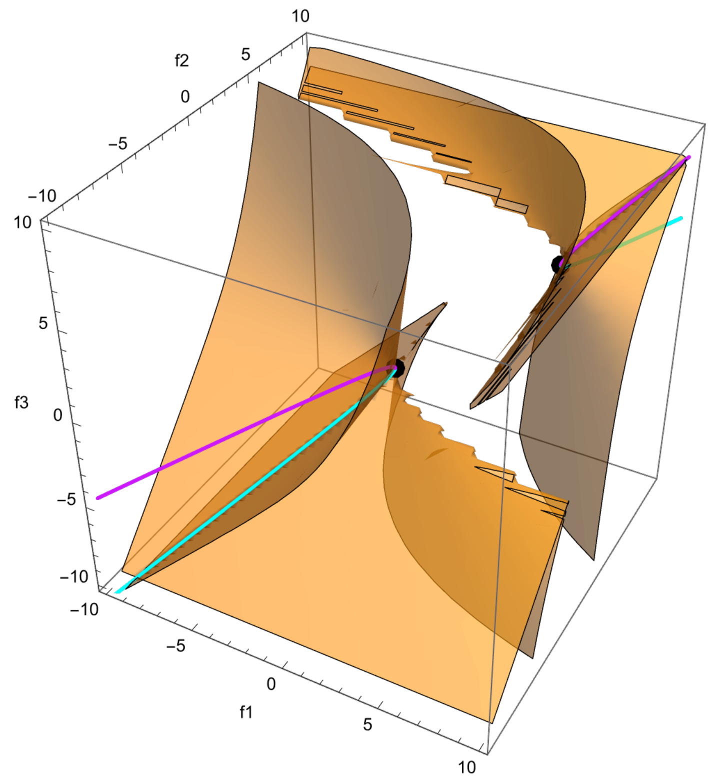
In[ ]:=

In[ ]:=
```
h1[f1_, f2_, f3_] := Discriminant[CharacteristicPolynomial[F[f1, f2, f3], ω], ω] ;
h2[f1_, f2_, f3_] := f1 - f2;
Solve[h1[f1, f2, f3] == 0 && h2[f1, f2, f3] == 0, {f2, f3}]
plot2 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 - Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0, 1, 1]];
plot3 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 + Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0.8, 0.1, 1]];
plot4 = Graphics3D[{PointSize[0.03],
    Point[{{3 + 2 Sqrt[2], 3 + 2 Sqrt[2], 2 + 3/2 Sqrt[2]}, {3 - 2 Sqrt[2], 3 - 2 Sqrt[2], 2 - 3/2 Sqrt[2]}}]}];
plot5 = Graphics3D[{PointSize[0.03], Red, Point[{3 + 2 Sqrt[2], 3 + 2 Sqrt[2], 2 + 3/2 Sqrt[2]}]}];
plot6 = Graphics3D[{PointSize[0.03], Blue, Point[{3 - 2 Sqrt[2], 3 - 2 Sqrt[2], 2 - 3/2 Sqrt[2]}]}];
Show[{plot1, plot2, plot3, plot4}]
Show[{plot1, plot2, plot3, plot5, plot6}]
```

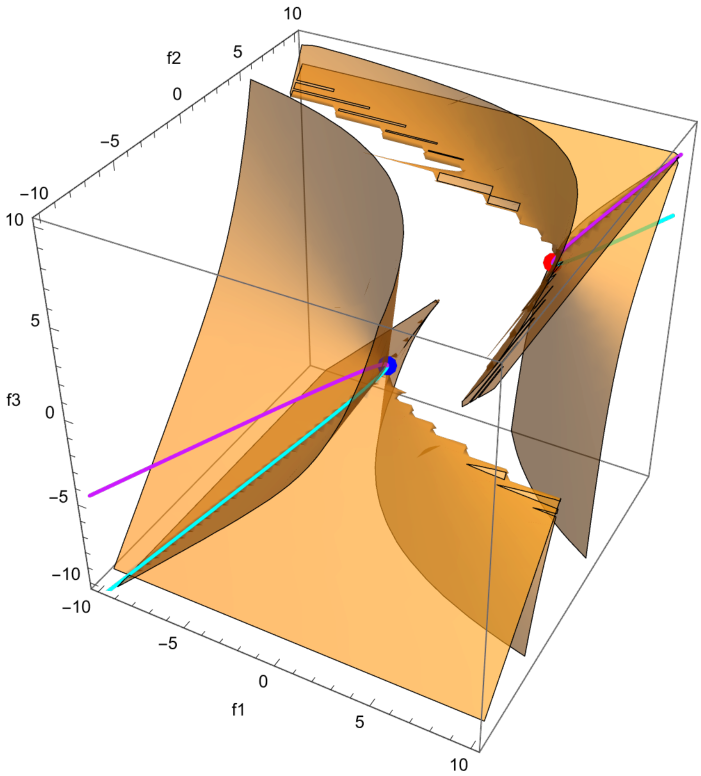
Out[ ]=

$$\left\{\left\{f2 \to f1,\ f3 \to \frac{1}{4}\left(-1 + 3\,f1 - \sqrt{1 - 6\,f1 + f1^2}\right)\right\},\ \left\{f2 \to f1,\ f3 \to \frac{1}{4}\left(-1 + 3\,f1 + \sqrt{1 - 6\,f1 + f1^2}\right)\right\}\right\}$$

Out[ ]=



Out[ ]=



In[ ]:=

$$\text{Eigenvalues}\left[F\left[3+2\sqrt{2},\,3+2\sqrt{2},\,2+\frac{3}{2}\sqrt{2}\right]\right]$$

$$\text{Eigenvectors}\left[F\left[3+2\sqrt{2},\,3+2\sqrt{2},\,2+\frac{3}{2}\sqrt{2}\right]\right]$$

$$\text{Eigenvalues}\left[F\left[3-2\sqrt{2},\,3-2\sqrt{2},\,2-\frac{3}{2}\sqrt{2}\right]\right]$$

$$\text{Eigenvectors}\left[F\left[3-2\sqrt{2},\,3-2\sqrt{2},\,2-\frac{3}{2}\sqrt{2}\right]\right]$$

Out[ ]=

$$\left\{-1-\sqrt{2},\,-1-\sqrt{2},\,-1-\sqrt{2}\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{-3-2\sqrt{2}}{4+3\sqrt{2}},\,0,\,1\right\},\,\left\{-\frac{-3-2\sqrt{2}}{4+3\sqrt{2}},\,1,\,0\right\},\,\{0,\,0,\,0\}\right\}$$

Out[ ]=

$$\left\{-1+\sqrt{2},\,-1+\sqrt{2},\,-1+\sqrt{2}\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{3-2\sqrt{2}}{-4+3\sqrt{2}},\,0,\,1\right\},\,\left\{-\frac{3-2\sqrt{2}}{-4+3\sqrt{2}},\,1,\,0\right\},\,\{0,\,0,\,0\}\right\}$$

In[ ]:=

$$\text{Eigenvalues}\left[F\left[f1, f1, \frac{1}{4}\left(-1+3\,f1-\sqrt{1-6\,f1+f1^2}\right)\right]\right]$$

$$\text{Eigenvectors}\left[F\left[f1, f1, \frac{1}{4}\left(-1+3\,f1-\sqrt{1-6\,f1+f1^2}\right)\right]\right]$$

$$\text{Plot}\left[\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}, \left\{f1, -1, 3-2\sqrt{2}\right\}, \text{PlotLegends} \to \text{"Expressions"}\right]$$

$$\text{Plot}\left[\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}, \left\{f1, 3+2\sqrt{2}, 7\right\}, \text{PlotLegends} \to \text{"Expressions"}\right]$$

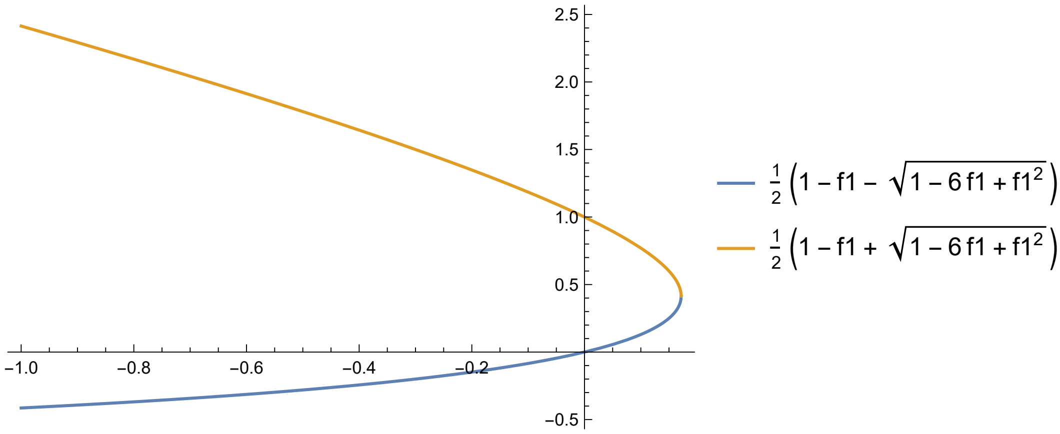
Out[ ]=

$$\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}$$

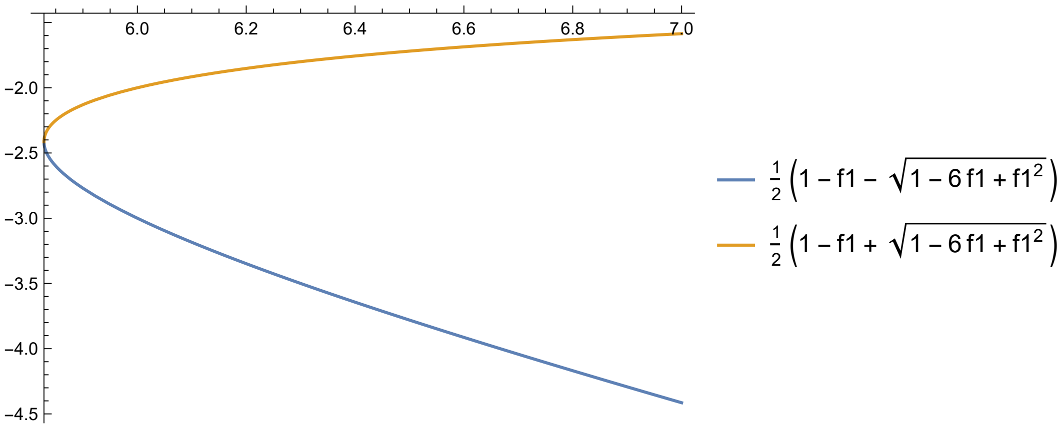
Out[ ]=

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1}, 0, 1\right\}, \left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1}, 1, 0\right\}, \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1}, 1, 1\right\}\right\}$$

Out[ ]=



Out[ ]=



In[ ]:=

$$\text{G1} = \text{Graphics3D}[\{\text{Opacity}[0.4], \text{Ball}[\,]\}];$$

$$\text{G2} = \text{ParametricPlot3D}\left[\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}, 0, \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\Big\},$$

$$\left\{f1,\ 0.1,\ 3-2\sqrt{2}\right\},\ \text{PlotStyle}\to\text{RGBColor}[1,\ 1,\ 0]\Big];$$

$$\text{G3}=\text{ParametricPlot3D}\Big[$$

$$\Big\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\Big\},$$

$$\left\{f1,\ 3-2\sqrt{2},\ 0.1\right\},\ \text{PlotStyle}\to\text{RGBColor}[1,\ 1,\ 0]\Big];$$

$$\text{Animate}\Big[\text{Show}\Big[$$

$$\Big\{\text{G1},\ \text{G2},\ \text{ParametricPlot3D}\Big[\Big\{\Big\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\right.$$

$$\left.\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\Big\}*u,\ \{u,\ 0,\ 1\}\Big]\ \text{/. Line}\to\text{Arrow}\Big\}\Big],$$

$$\left\{f1,\ 0.1,\ 3-2\sqrt{2}\right\},\ \text{AnimationRunning}\to\text{False}\Big]$$

$$\text{Animate}\Big[\text{Show}\Big[\Big\{\text{G1},\ \text{G3},\ \text{ParametricPlot3D}\Big[$$

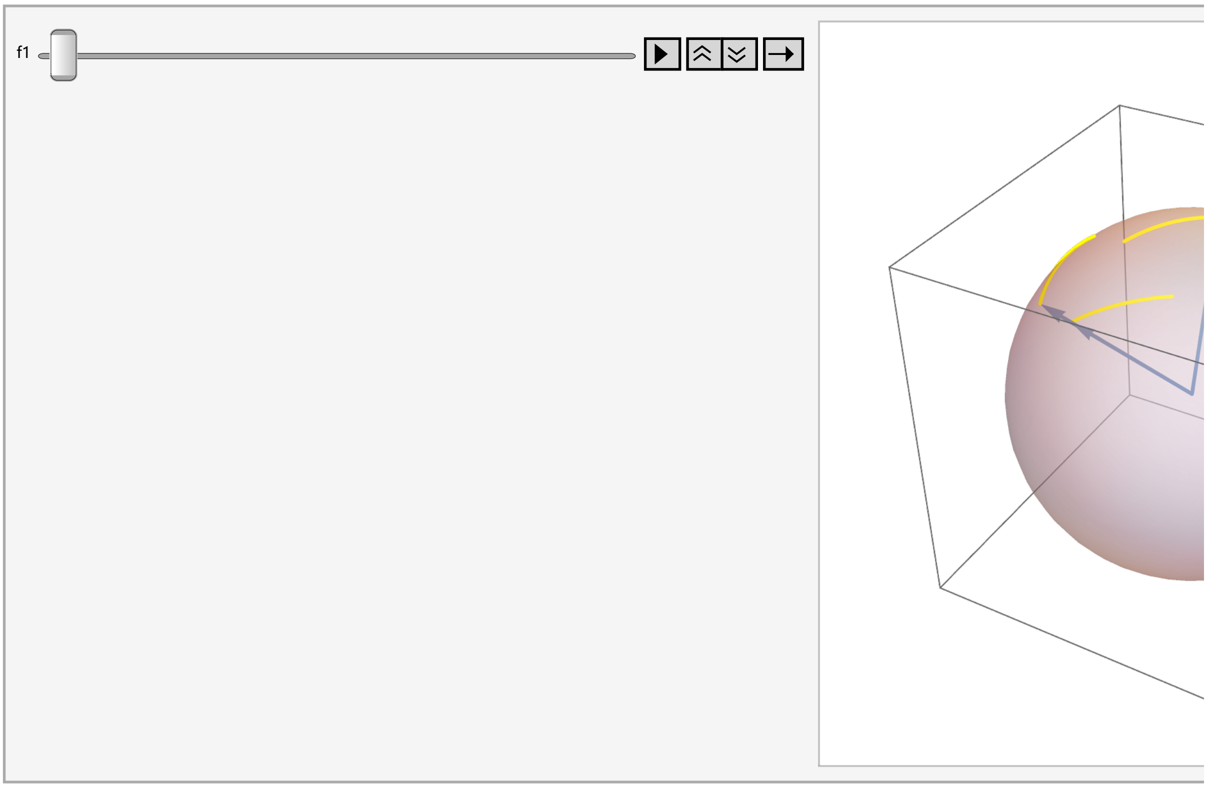
$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

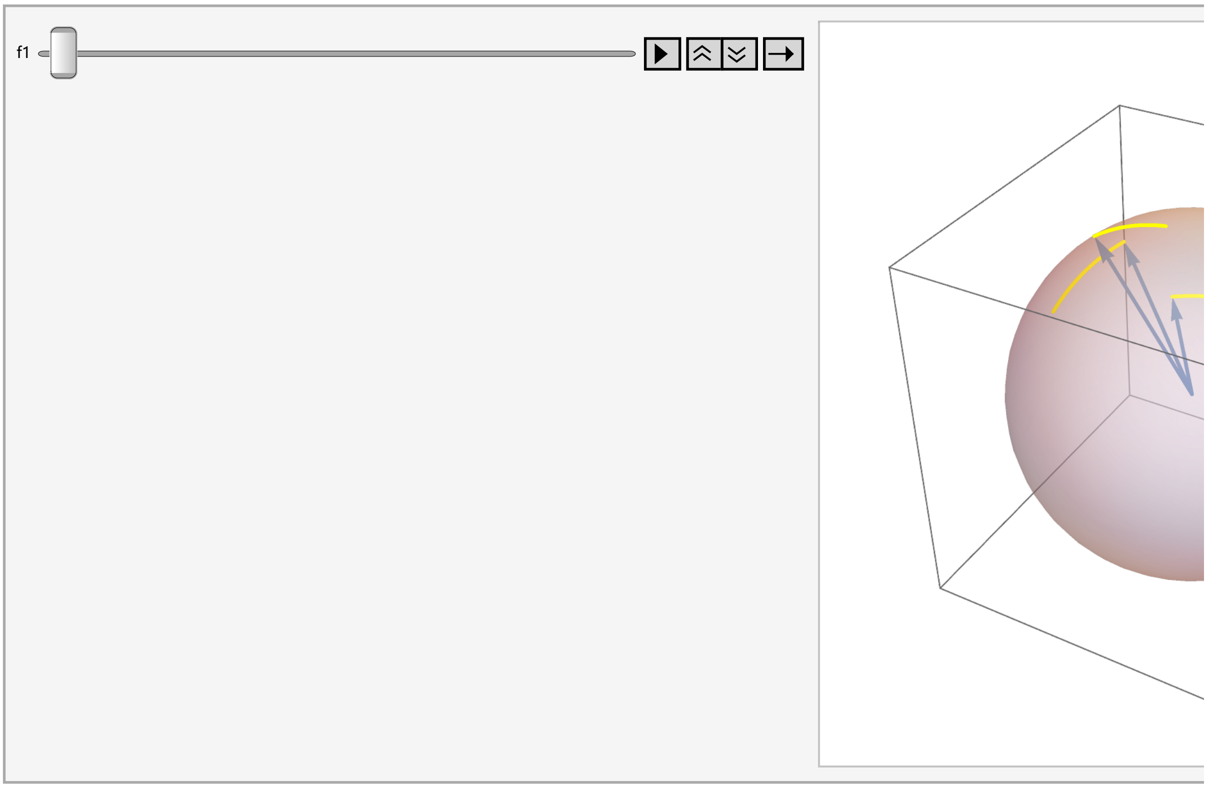
$$\left.\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\right\} * u,\ \{u,\ 0,\ 1\}\Big]\ /.$$

Line → Arrow}], {f1, 3 - 2 $\sqrt{2}$, 0.1}, AnimationRunning → False]

Out[ ]=



Out[ ]=



In[ ]:= $\text{Eigenvalues}\left[F\left[f1, f1, \frac{1}{4}\left(-1+3\,f1+\sqrt{1-6\,f1+f1^2}\right)\right]\right]$

$\text{Eigenvectors}\left[F\left[f1, f1, \frac{1}{4}\left(-1+3\,f1+\sqrt{1-6\,f1+f1^2}\right)\right]\right]$

Out[ ]=

$$\left\{\frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1-\sqrt{1-6\,f1+f1^2}\right), \frac{1}{2}\left(1-f1+\sqrt{1-6\,f1+f1^2}\right)\right\}$$

Out[ ]=

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1}, 1, 1\right\}, \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1}, 0, 1\right\}, \left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1}, 1, 0\right\}\right\}$$

In[ ]:= $\text{Eigenvalues}\left[F\left[0, 0, -\frac{1}{2}\right]\right]$

$\text{Eigenvectors}\left[F\left[0, 0, -\frac{1}{2}\right]\right]$

Out[ ]=

{1, 1, 0}

Out[ ]=

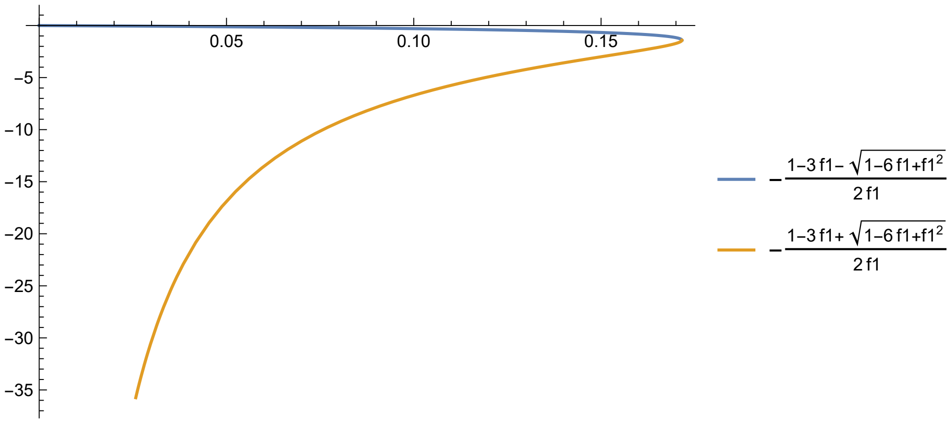
{{0, -1, 1}, {1, 0, 0}, {0, 1, 1}}

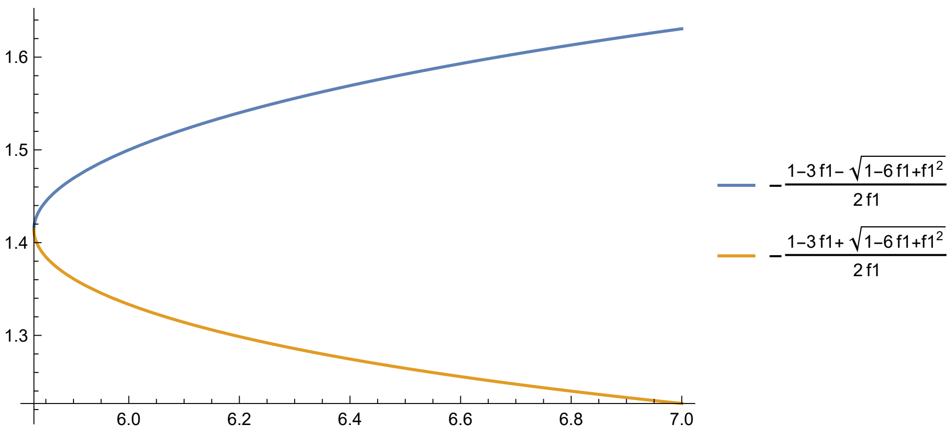
In[ ]:=
```
Eigenvalues[F[0, 0, 0]]
Eigenvectors[F[0, 0, 0]]
```

Out[ ]=
{1, 0, 0}

Out[ ]=
{{1, 0, 0}, {0, 0, 1}, {0, 1, 0}}

In[ ]:=
$$\text{Plot}\left[\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1},\ -\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1}\right\},\ \left\{f1,\ 0,\ 3-2\,\sqrt{2}\right\},\ \text{PlotLegends}\to\text{"Expressions"}\right]$$
$$\text{Plot}\left[\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1},\ -\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1}\right\},\ \left\{f1,\ 3+2\,\sqrt{2},\ 7\right\},\ \text{PlotLegends}\to\text{"Expressions"}\right]$$

Out[ ]=


Out[ ]=


In[ ]:=
$$\text{G4} = \text{ParametricPlot3D}\left[\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},\right.$$
$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\Big\},$$

$$\left\{f1,\ 20,\ 3+2\,\sqrt{2}\right\},\ \text{PlotStyle}\to\text{RGBColor}[1,\ 1,\ 0]\Big];$$

G5 = ParametricPlot3D[

$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left.\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\right\},$$

$$\left\{f1,\ 3+2\,\sqrt{2},\ 20\right\},\ \text{PlotStyle}\to\text{RGBColor}[1,\ 1,\ 0]\Big];$$

Animate[Show[

$$\Big\{G1,\ G4,\ \text{ParametricPlot3D}\Big[\Big\{\Big\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

$$\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{4\,f1\,\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\},$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\right.$$

$$\left.\frac{1}{\sqrt{2+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\}\Big\}*u,\ \{u,\ 0,\ 1\}\Big]\ /.\ \text{Line}\to\text{Arrow}\Big\}\Big],$$

$$\left\{f1,\ 20,\ 3+2\,\sqrt{2}\right\},\ \text{AnimationRunning}\to\text{False}\Big]$$

Animate[Show[{G1, G5, ParametricPlot3D[

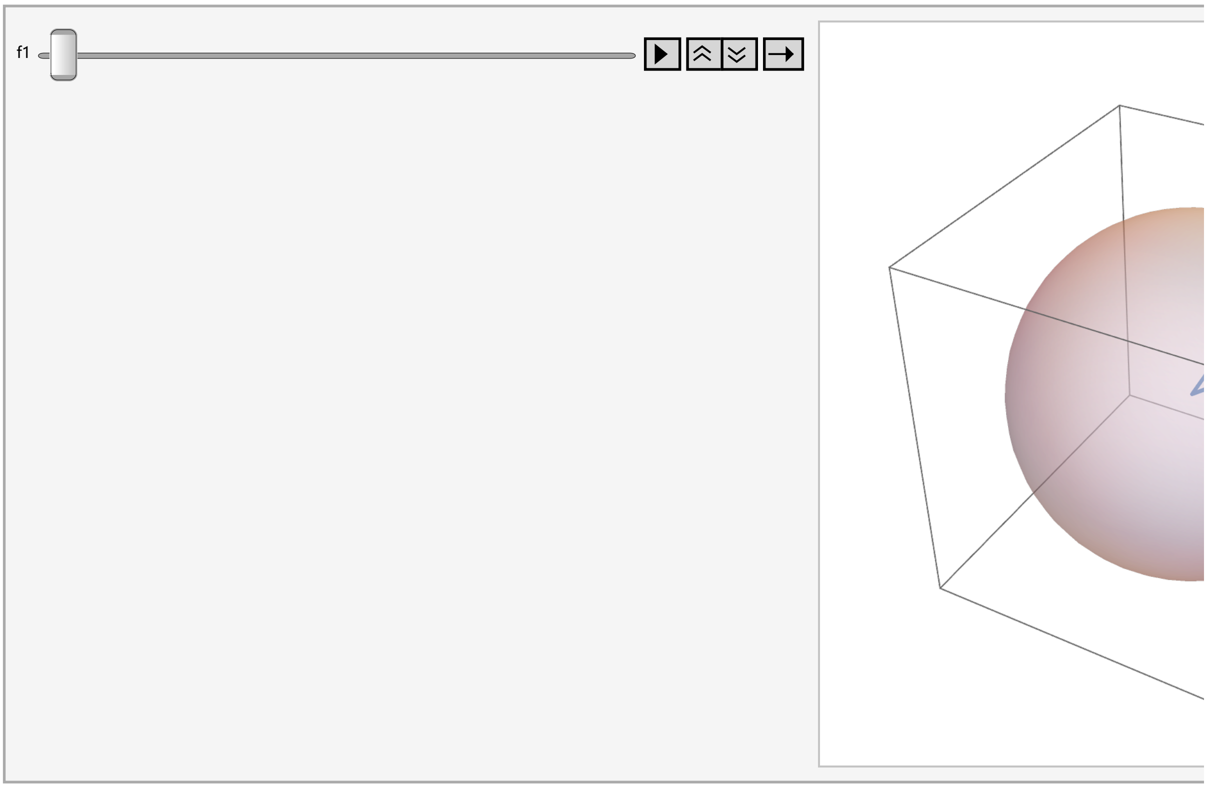
$$\left\{\left\{-\frac{1-3\,f1+\sqrt{1-6\,f1+f1^2}}{2\,f1\,\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}},\ \frac{1}{\sqrt{2+\frac{\left(1-3\,f1+\sqrt{1-6\,f1+f1^2}\right)^2}{4\,f1^2}}}\right\},\right.$$

$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0,\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}}\right\},$$

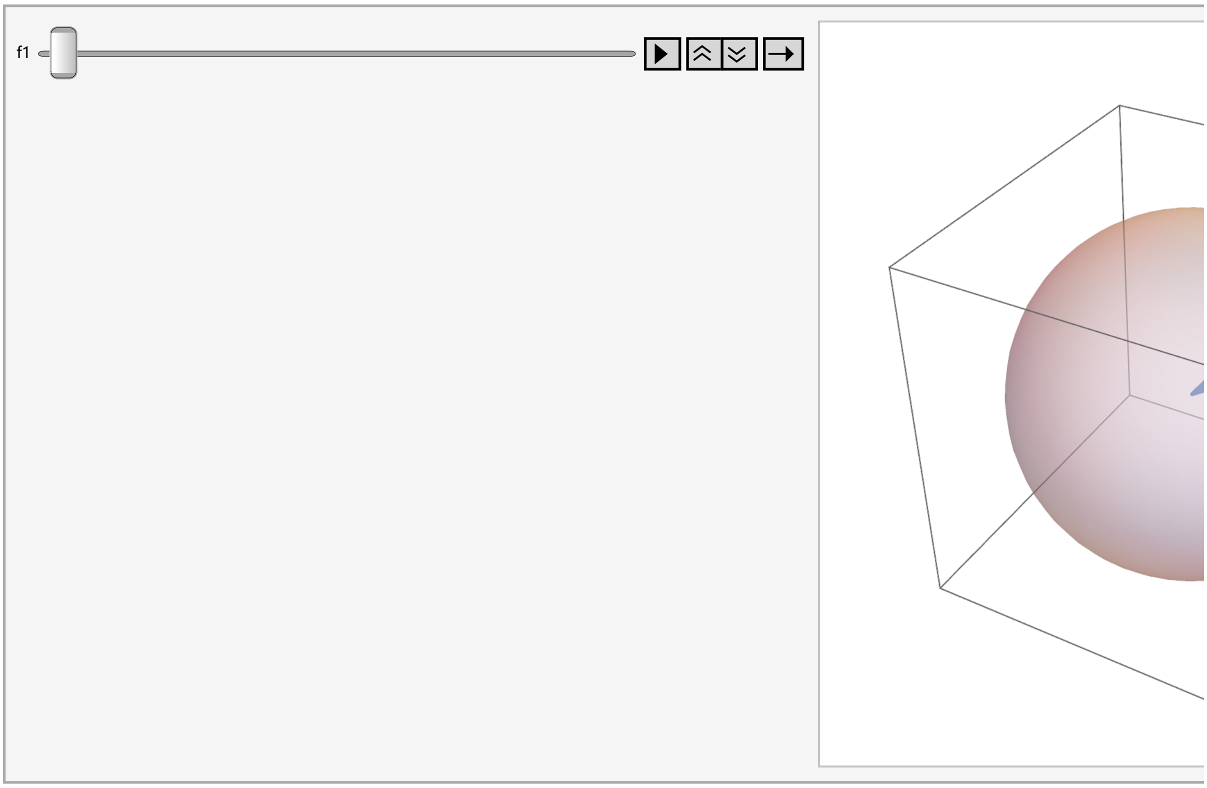
$$\left\{-\frac{1-3\,f1-\sqrt{1-6\,f1+f1^2}}{4\,f1\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ \frac{1}{\sqrt{1+\frac{\left(1-3\,f1-\sqrt{1-6\,f1+f1^2}\right)^2}{16\,f1^2}}},\ 0\right\}\right\} * u,\ \{u,0,1\}\Big]\ /.$$

```
Line → Arrow}], {f1, 3 + 2 √2 , 20}, AnimationRunning → False]
```

Out[ ]=



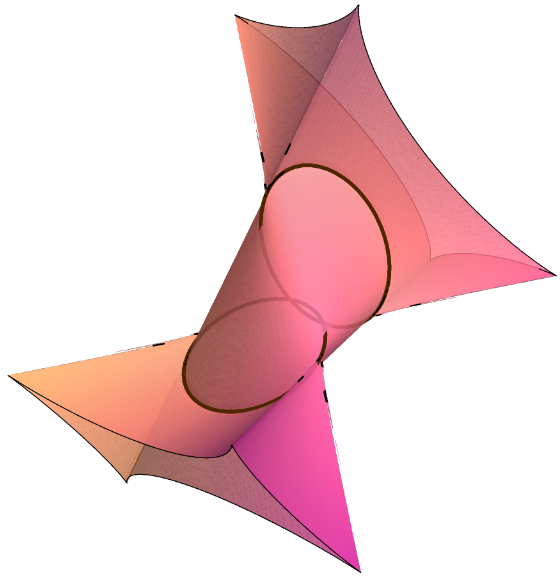
Out[ ]=



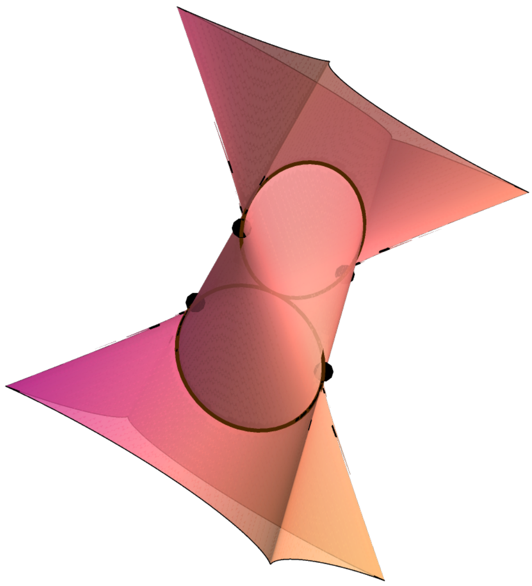
In[ ]:=

```
(*This is for visualize sw4*)
H[f1_, f2_, f3_] := ( 2   f1 f2
                     -f1  0  f3
                     -f2 f3  0 )
s = 3;
(*g is the discriminant surface*)
g = ContourPlot3D[Discriminant[CharacteristicPolynomial[H[f1, f2, f3], ω], ω] == 0,
   {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, AxesLabel → Automatic,
   Boxed → False, Axes → False, Mesh → None, PlotPoints → 80,
ColorFunction → (Blend[{Purple, Pink, Lighter@Orange}, Mean[{#1, #2}]] &),
ContourStyle → Directive[Opacity[.9]]];

(*f1 and f2 are NLs*)
f1 = ParametricPlot3D[{Cos[t], -Cos[t], Sin[t] + 1},
   {t, 0, 2 π}, PlotStyle → RGBColor[0.30196, 0.14902, 0.00000]];
f2 = ParametricPlot3D[{Cos[t], Cos[t], Sin[t] - 1},
   {t, 0, 2 π}, PlotStyle → RGBColor[0.30196, 0.14902, 0.00000]];

(*p are MPs, *)
p = Graphics3D[{PointSize[0.03], Point[{{2 √2/3, -2 √2/3, 2/3},
     {-2 √2/3, 2 √2/3, 2/3}, {-2 √2/3, -2 √2/3, -2/3}, {2 √2/3, 2 √2/3, -2/3}}]}];

Show[{g, f1, f2 }]
Show[{g, f1, f2 , p}]
```

Out[ ]=

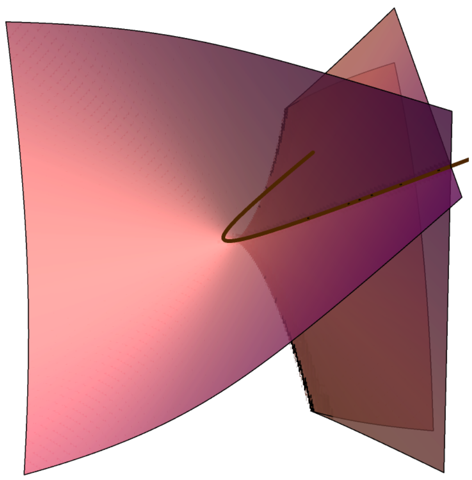


Out[ ]=



In[ ]:=
```
(*This is for visualize sw2*)
G[f1_, f2_, f3_] := ( 1 - f1 - f2    f1       f2
                      -f1            f1 - f3  f3
                      -f2            f3       f2 - f3 )
s = 3;
(*g is the discriminant surface*)
g = ContourPlot3D[Discriminant[CharacteristicPolynomial[G[f1, f2, f3], ω], ω] == 0,
   {f1, -s, s}, {f2, -s, s}, {f3, -s, s}, AxesLabel → Automatic,
   Boxed → False, Axes → False, Mesh → None, PlotPoints → 80,
ColorFunction → (Blend[{Purple, Pink, Lighter@Orange}, Mean[{#1, #2}]] &),
ContourStyle → Directive[Opacity[.9]]];

(*plot1 and plot2 are NL and NIL, respectively. They are ploted in the same color*)
plot1 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 + Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0.30196, 0.14902, 0.00000]];
plot2 = ParametricPlot3D[{f1, f1, 1/4 (-1 + 3 f1 - Sqrt[1 - 6 f1 + f1^2])},
   {f1, -s, s}, PlotStyle → RGBColor[0.30196, 0.14902, 0.00000]];

(*p1 and p2 are the MPs*)
p1 = Graphics3D[{PointSize[0.03], Point[{3 + 2 Sqrt[2], 3 + 2 Sqrt[2], 2 + 3/2 Sqrt[2]}]}];
p2 = Graphics3D[{PointSize[0.03], Point[{3 - 2 Sqrt[2], 3 - 2 Sqrt[2], 2 - 3/2 Sqrt[2]}]}];

Show[{g, plot1, plot2}]
Show[{g, plot1, plot2, p1, p2}]
```

Out[ ]=



Out[ ]=